Eclipse LB

Technical Reference Manual

9231659A 12/99

Canberra Industries, 800 Research Parkway, Meriden, CT 06450
Tel: 203-238-2351 FAX: 203-235-1347 http://www.canberra.com

Printed in the United States of America.

Table of Contents

1. Introduction

This document provides technical information about the architecture and processing of data in the Eclipse Software, along with a description of the algorithms and calculations used in Eclipse.

1.1 Relevant Documents

1. *Eclipse LB User's Manual*.

2. Plateau

The optimum operating voltage is determined from the plateau curve as the leftmost point that satisfies the following two conditions:

1. The "slope" (in % per 100 volts - as defined below), %M, is less than 2.5% (per 100 volts).
2. The number of counts observed for this point is greater than 2500 (for 2% counting statistics).

For a set of N data points (x_i, y_i), the coefficients of the best fit straight line (y=mx+b) by the method of least squares are given by

$$m = \frac{N \cdot \sum_{i=1}^{N} x_i \cdot y_i - \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} y_i}{N \cdot \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i \right)^2}$$

$$b = \frac{\sum_{i=1}^{N} x_i^2 \cdot \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} x_i y_i}{N \cdot \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i \right)^2}$$

In the case of the plateau determination, the data consists of a set of points (V_j, C_j), where C_j is the number of counts observed during a given counting interval when a voltage of V_j is applied to the detector. The slope (in counts per volt) at the i^{th} point, m_i, is defined as the slope of the straight line (C=mV+b) determined from the 5 points about the i^{th} point (i.e., from i-2 to i+2) by the method of least squares.

By analogy to the general case above, we write:

$$m_i = \frac{5 \cdot \sum_{j=i-2}^{i+2} V_j \cdot C_j - \sum_{j=i-2}^{i+2} V_j \cdot \sum_{j=i-2}^{i+2} C_j}{5 \cdot \sum_{j=i-2}^{i+2} V_j^2 - \left(\sum_{j=i-2}^{i+2} V_j \right)^2}$$

The "percent slope" at the i^{th} point, $\%m_i$, can be written as

$$\%m_i = \frac{m_i}{C_i} \cdot 100\%$$

and the "percent slope per 100 volts" at the i^{th} point, $\%M_i$, as

$$\%M_i = \%m_i \cdot 100(volts) = \frac{m_i}{C_i} \cdot 100\% \cdot 100 = \frac{m_i}{C_i} \cdot 10000\%$$

$$\%M_i = \frac{\left(\frac{5 \cdot \sum_{j=i-2}^{i+2} V_j \cdot C_j - \sum_{j=i-2}^{i+2} V_j \cdot \sum_{j=i-2}^{i+2} C_j}{5 \cdot \sum_{j=i-2}^{i+2} V_j^2 - \left(\sum_{j=i-2}^{i+2} V_j \right)^2} \right)}{C_i} \cdot 10000\%$$

In summary, the operating voltage is selected as the leftmost point (lowest i) for which

1. $\%M_i < 2.5\%$
2. $C_i > 2500$.

3. Background

Backgrounds may be determined for each of the three count modes:

1. Alpha only
2. Simultaneous
3. Alpha then Beta

Backgrounds are always determined as a gross count, and may be determined from a single measurement or a set of measurements; e.g., 1 – 10 minute count or 10 – 1 minute counts.

3.1 Alpha Only Mode

Rates

For $N \geq 1$:

$$R_B = \frac{1}{N} \cdot \sum_{i=1}^{N} \left[\frac{C_i}{T} \right] = \frac{1}{N} \cdot \sum_{i=1}^{N} R_{B\_i}$$

$R_B =$ the BACKGROUND count rate.

$C_i =$ the number of counts obtained during the ith count.

$T =$ the (common) count time of each of the N counts.

Uncertainties:

For $N = 1$:

$$\sigma_{R_B} = \sqrt{\left(\frac{R_B}{T} \right)}$$

$\sigma_{R_B} =$ the **uncertainty** in the BACKGROUND count rate.

For $N > 1$:

$$\sigma_{R_B}^2 = \frac{1}{(N-1)} \cdot \sum_{i=1}^{N} \left(R_{B\_i} - R_B \right)^2$$

or

$$\sigma_{R_B} = \sqrt{\frac{\sum_{i=1}^{N} \left(R_{B\_i} - R_B \right)^2}{(N-1)}}$$

3.2 Simultaneous Mode

Rates

For $N \geq 1$:

$$R_B = \frac{1}{N} \cdot \sum_{i=1}^{N} \left[\frac{C_i}{T} \right] = \frac{1}{N} \cdot \sum_{i=1}^{N} R_{B\_i}$$

$R_B =$ the BACKGROUND count rate.

$C_i =$ the number of counts obtained during the ith count.

$T =$ the (common) count time of each of the *N* counts.

Uncertainties:

For $N = 1$:

$$\sigma_{R_B} = \sqrt{\left(\frac{R_B}{T} \right)}$$

$\sigma_{R_B} =$ the **uncertainty** in the BACKGROUND count rate.

For $N > 1$:

$$\sigma_{R_B}^2 = \frac{1}{(N-1)} \cdot \sum_{i=1}^{N} \left(R_{B\_i} - R_B \right)^2$$

or

$$\sigma_{R_B} = \sqrt{\frac{\sum_{i=1}^{N} \left(R_{B\_i} - R_B \right)^2}{(N-1)}}$$

3.3 Alpha Then Beta Mode

Since the ATB mode matches one α only count (i) to a corresponding $[\alpha+\beta]$ count (i), and the count times are the same, we may write:

$$T_{[\alpha+\beta]\_i} = T_{\alpha\_i} = T$$

Alpha Background in the ATB Mode

The Alpha Background Rate is given by

For $N \geq 1$

$$R_{B\_\alpha} = \frac{1}{N} \cdot \sum_{i=1}^{N} \left[\frac{C_{\alpha\_i}}{T} \right] = \frac{1}{N} \cdot \sum_{i=1}^{N} R_{B\_\alpha\_i}$$

The corresponding uncertainty is then given by

For $N = 1$:

$$\sigma_{R_{B\_\alpha}} = \sqrt{\left(\frac{R_{B\_\alpha}}{T} \right)}$$

For $N > 1$:

$$\sigma^2_{R_{B\_\alpha}} = \frac{1}{(N-1)} \cdot \sum_{i=1}^{N} \left(R_{B\_\alpha\_i} - R_{B\_\alpha} \right)^2$$

or

$$\sigma_{R_{B\_\alpha}} = \sqrt{\frac{\sum_{i=1}^{N} \left(R_{B\_\alpha\_i} - R_{B\_\alpha} \right)^2}{(N-1)}}$$

where,

$R_{B\_\alpha} =$ the α background count rate for the α ONLY mode of the ATB count mode.

$C_{\alpha\_i} =$ the number of α counts obtained during the ith count.

$T =$ the (common) count time of each of the N counts.

$R_{B\_\alpha\_i} =$ the α background count rate determined during the ith count.

$\sigma_{R_{B\_\alpha}} =$ the **uncertainty** in the α (system) BACKGROUND count rate for the α ONLY mode of the ATB count mode.

Beta Background in the ATB Mode

The Beta Background Rate is given by

For $N \geq 1$

$$R_{B\_\beta} = \frac{1}{N} \cdot \sum_{i=1}^{N} \left[R_{B\_[\alpha+\beta]\_i} - R_{B\_\alpha\_i} \right] = R_{B\_[\alpha+\beta]} - R_{B\_\alpha}$$

where,

$R_{B\_\beta} =$ the **derived** β BACKGROUND count rate for the ATB count mode.

$R_{B\_[\alpha+\beta]\_i} =$ the GROSS ($\alpha + \beta$) count rate obtained in the ($\alpha + \beta$) mode during the i^{th} count.

$R_{B\_\alpha\_i} =$ the GROSS (α) count rate obtained in the α ONLY mode during the i^{th} count.

$R_{B\_[\alpha+\beta]} = \frac{1}{N} \cdot \sum_{i=1}^{N} R_{B\_[\alpha+\beta]\_i} =$ the ($\alpha + \beta$) BACKGROUND count rate for the ($\alpha + \beta$) mode of the ATB count mode.

$R_{B\_\alpha} =$ the α BACKGROUND count rate for the α ONLY mode of the ATB count mode.

4. Count Rates (Other than Background)

The remaining functions (Efficiency and Activity) depend on the corrected count rate of a standard (for efficiency) or a sample (for activity determinations). While the background count rate can be determined as an average from a set of N measurements, all other count rates are determined from a single measurement[1]. These count rates are determined as follows:

4.1 Alpha Only Mode

When Method Blank subtraction is implemented, the blank is counted as part of the batch, and the count times for the sample and blank are the same:

$$T_1 = T_{blank} = T$$

Rate:

$$R = \frac{C_1}{T_1} - \delta_1 \cdot R_{B\_\alpha} - \delta_2 \cdot \left(\frac{C_{blank}}{T_1} - \delta_1 \cdot R_{B\_\alpha}\right)$$

$$= R_1 - \delta_1 \cdot R_{B\_\alpha} - \delta_2 \cdot (R_{blank\_gross} - \delta_1 \cdot R_{B\_\alpha})$$

$$= R_1 - \delta_1 \cdot R_{B\_\alpha} - \delta_2 \cdot R_{blank\_gross} + \delta_1 \cdot \delta_2 \cdot R_{B\_\alpha}$$

$$= R_1 - \delta_2 \cdot R_{blank\_gross} - \delta_1 \cdot R_{B\_\alpha} + \delta_1 \cdot \delta_2 \cdot R_{B\_\alpha}$$

$$= R_1 - \delta_2 \cdot R_{blank\_gross} - \delta_1 \cdot [1 - \delta_2] \cdot R_{B\_\alpha}$$

$C_1 =$ the number of counts obtained during this (one) measurement.

$T_1 =$ the count time of this measurement.

$R_1 = \frac{C_1}{T_1} =$ the gross count rate of the sample (or standard in the case of efficiency).

$R_{B\_\alpha} =$ the BACKGROUND count rate for the Alpha Only count mode.

$C_{blank} =$ the number of counts obtained during this (one) measurement of the (one and only one) designated blank.

$R_{blank\_gross} = \frac{C_{blank}}{T_1} =$ the GROSS count rate of the (one and only one) designated blank.

$$\delta_1 = \begin{Bmatrix} 0\_if\_BACKGROUND\_SUBTRACTION = NO \\ 1\_if\_BACKGROUND\_SUBTRACTION = YES \end{Bmatrix}$$

$$\delta_2 = \begin{Bmatrix} 0\_if\_BLANK\_SUBTRACTION = NO \\ 1\_if\_BLANK\_SUBTRACTION = YES \end{Bmatrix}$$

[1] The efficiency can also be determined as an average from a set of *N* measurements; however, this is an average of *N* efficiency measurements – each determined from a single measurement of the count rate of the calibration standard.

Note: $\delta_1 = \delta_2 = 1$ IS NOT CURRENTLY ALLOWED

Uncertainties:

$$\sigma_R = \sqrt{\left(\frac{R_1}{T_1}\right) + \delta_2^2 \cdot \sigma_{blank\_gross}^2 + \delta_1^2 \cdot [1 - \delta_2]^2 \cdot \sigma_{R_{B\_\alpha}}^2}$$

$$\sigma_R = \sqrt{\left(\frac{R_1}{T}\right) + \delta_2^2 \cdot \left(\frac{R_{blank\_gross}}{T}\right) + \delta_1^2 \cdot [1 - \delta_2]^2 \cdot \sigma_{R_{B\_\alpha}}^2}$$

$\sigma_{R_{B\_\alpha}} =$ the **uncertainty** in the Alpha Only (system) BACKGROUND count rate.

$= \sqrt{\dfrac{R_{B\_\alpha}}{T_B}}$ for the case in which the background was determined from a single measurement

$= \sqrt{\dfrac{\sum_{i=1}^{N} \left(R_{B\_\alpha\_i} - R_{B\_\alpha}\right)^2}{(N-1)}}$ if the background was determined from a set of N measurements.

$$\delta_1 = \begin{Bmatrix} 0\_if\_BACKGROUND\_SUBTRACTION = NO \\ 1\_if\_BACKGROUND\_SUBTRACTION = YES \end{Bmatrix}$$

$$\delta_2 = \begin{Bmatrix} 0\_if\_BLANK\_SUBTRACTION = NO \\ 1\_if\_BLANK\_SUBTRACTION = YES \end{Bmatrix}$$

Note: $\delta_1 = \delta_2 = 1$ IS NOT CURRENTLY ALLOWED

4.2 Simultaneous Mode

When Method Blank subtraction is implemented, the blank is counted as part of the batch, and the count times for the sample and blank are the same:

$$T_1 = T_{blank} = T$$

Without Spillover Correction

The equations for the count rate and corresponding uncertainty for the simultaneous mode without spillover correction are identical to those for alpha only, except that they are applied to each channel (of the simultaneous mode) individually:

Rate:

$$R = \frac{C_1}{T_1} - \delta_1 \cdot R_B - \delta_2 \cdot (\frac{C_{blank}}{T_1} - \delta_1 \cdot R_B)$$

$$= R_1 - \delta_2 \cdot R_{blank\_gross} - \delta_1 \cdot [1 - \delta_2] \cdot R_B$$

$C_1 =$ the number of counts obtained (in the channel of interest) during this (one) measurement.

$T_1 =$ the count time of this measurement.

$R_1 = \frac{C_1}{T_1} =$ the gross count rate of the sample.

$R_B =$ the BACKGROUND count rate for the channel of interest for the Simultaneous count mode. (i.e., R_{B_α} for alpha and R_{B_β} for beta.)

$R_{blank\_gross} =$ the GROSS count rate for the channel of interest of the (one and only one) designated blank.

$\delta_1 = \begin{Bmatrix} 0\_if\_BACKGROUND\_SUBTRACTION = NO \\ 1\_if\_BACKGROUND\_SUBTRACTION = YES \end{Bmatrix}$

$\delta_2 = \begin{Bmatrix} 0\_if\_BLANK\_SUBTRACTION = NO \\ 1\_if\_BLANK\_SUBTRACTION = YES \end{Bmatrix}$

Note: $\delta_1 = \delta_2 = 1$ IS NOT CURRENTLY ALLOWED

Uncertainties:

$$\sigma_R = \sqrt{\left(\frac{R_1}{T_1}\right) + \delta_2^2 \cdot \sigma_{blank\_gross}^2 + \delta_1^2 \cdot [1 - \delta_2]^2 \cdot \sigma_{R_B}^2}$$

$$\sigma_R = \sqrt{\left(\frac{R_1}{T}\right) + \delta_2^2 \cdot \left(\frac{R_{blank\_gross}}{T}\right) + \delta_1^2 \cdot [1 - \delta_2]^2 \cdot \sigma_{R_B}^2}$$

$\sigma_{R_B} =$ the **uncertainty** in the (system) BACKGROUND count rate for the channel of interest for the Simultaneous count mode.

$$= \sqrt{\frac{R_B}{T_B}}$$ for the case in which the background was determined from a single measurement

$$= \sqrt{\frac{\sum_{i=1}^{N}\left(R_{B\_i} - R_B\right)^2}{(N-1)}}$$ if the background was determined from a set of N measurements.

$R_{B\_i} = \ R_{B\_\alpha\_i}$ if the α channel is the channel of interest

$= \ R_{B\_\beta\_i}$ if the β channel is the channel of interest

$$\delta_1 = \begin{Bmatrix} 0\_if\_BACKGROUND\_SUBTRACTION = NO \\ 1\_if\_BACKGROUND\_SUBTRACTION = YES \end{Bmatrix}$$

$$\delta_2 = \begin{Bmatrix} 0\_if\_BLANK\_SUBTRACTION = NO \\ 1\_if\_BLANK\_SUBTRACTION = YES \end{Bmatrix}$$

Note: $\delta_1 = \delta_2 = 1$ IS NOT CURRENTLY ALLOWED

With Spillover Correction

The equations for the count rate and corresponding uncertainty for the simultaneous mode with spillover correction are derived in Appendix A. The results are presented below:

Rate:

The spillover corrected count rates can be written as

$$R'_\beta = R_{\beta\_corrected} = \frac{\left\lfloor\left(R_\beta - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_\beta \cdot (1-\delta_2)\right) - \left(R_\alpha - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_\alpha (1-\delta_2)\right) \cdot \chi_{\alpha->\beta}\right\rfloor}{[1-\chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}]}$$

$$R'_\alpha = R_{\alpha\_corrected} = \frac{\left\lfloor\left(R_\alpha - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_\alpha (1-\delta_2)\right) - \left(R_\beta - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_\beta \cdot (1-\delta_2)\right) \cdot \chi_{\beta->\alpha}\right\rfloor}{[1-\chi_{\beta->\alpha} \cdot \chi_{\alpha->\beta}]}$$

Uncertainties:

The uncertainty in the spillover corrected count rates can be written as

$$\sigma^2_{R'_\alpha} = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\alpha} + (\delta_2)^2\cdot\sigma^2_{M_{\alpha\_gross}} + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\alpha}\right]$$

$$+ \left[\frac{\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\beta} + (\delta_2)^2\cdot\sigma^2_{M_{\beta\_gross}} + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\beta}\right]$$

$$+ \left[\frac{-R'_\beta\cdot\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\beta->\alpha}}}{\chi_{\beta->\alpha}}\right)^2$$

$$+ \left[\frac{R'_\alpha\cdot\chi_{\beta->\alpha}\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\alpha->\beta}}}{\chi_{\alpha->\beta}}\right)^2$$

$$\sigma^2_{R'_\beta} = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\beta} + (\delta_2)^2\cdot\sigma^2_{M_{\beta\_gross}} + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\beta}\right]$$

$$+ \left[\frac{\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\alpha} + (\delta_2)^2\cdot\sigma^2_{M_{\alpha\_gross}} + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\alpha}\right]$$

$$+ \left[\frac{-R'_\alpha\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\alpha->\beta}}}{\chi_{\alpha->\beta}}\right)^2$$

$$+ \left[\frac{R'_\beta\cdot\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\beta->\alpha}}}{\chi_{\beta->\alpha}}\right)^2$$

4.3 Alpha then Beta Mode

Once again, since only one blank per batch is allowed, and the blank is counted as part of the batch, the count times for the sample and blank are the same:

$$T_1 = T_{blank} = T$$

Alpha Count Rate in the ATB Mode:

$$R_\alpha = R_{\alpha\_gross} - \delta_2 \cdot R_{\alpha\_blank\_gross} - \delta_1 \cdot [1-\delta_2] \cdot R_{B\_\alpha}$$

R_{α_gross} = the gross α count rate obtained during the α ONLY count of the ATB count.

$R_{\alpha\_blank\_gross}$ = the GROSS α count rate obtained during the α ONLY count of the ATB count of the (one and only one) designated blank.

R_{B_α} = the α (system) background count rate for the α ONLY mode of the ATB count mode.

$$\delta_1 = \begin{Bmatrix} 0\_if\_BACKGROUND\_SUBTRACTION = NO \\ 1\_if\_BACKGROUND\_SUBTRACTION = YES \end{Bmatrix}$$

$$\delta_2 = \begin{Bmatrix} 0\_if\_BLANK\_SUBTRACTION = NO \\ 1\_if\_BLANK\_SUBTRACTION = YES \end{Bmatrix}$$

Note: $\delta_1 = \delta_2 = 1$ IS NOT CURRENTLY ALLOWED

Uncertainties:

$$\sigma_{R_\alpha} = \sqrt{\left(\frac{R_{\alpha\_gross\_1}}{T_1}\right) + \delta_2^2 \cdot \sigma^2_{\alpha\_blank\_gross} + \delta_1^2 \cdot [1-\delta_2]^2 \cdot \sigma^2_{R_{B\_\alpha}}}$$

$$\sigma_{R_\alpha} = \sqrt{\left(\frac{R_{\alpha\_gross\_1}}{T}\right) + \delta_2^2 \cdot \left(\frac{R_{\alpha\_blank\_gross}}{T}\right) + \delta_1^2 \cdot [1-\delta_2]^2 \cdot \sigma^2_{R_{B\_\alpha}}}$$

$\sigma_{R_{B\_\alpha}}$ = the **uncertainty** in the α (system) BACKGROUND count rate for the α ONLY mode

$= \sqrt{\frac{R_{B\_\alpha}}{T_B}}$ for the case in which the background was determined from a single measurement

$= \sqrt{\frac{\sum_{i=1}^{N}\left(R_{B\_\alpha\_i} - R_{B\_\alpha}\right)^2}{(N-1)}}$ if the background was determined from a set of N measurements.

$$\delta_1 = \begin{Bmatrix} 0\_if\_BACKGROUND\_SUBTRACTION = NO \\ 1\_if\_BACKGROUND\_SUBTRACTION = YES \end{Bmatrix}$$

$$\delta_2 = \begin{Bmatrix} 0\_if\_BLANK\_SUBTRACTION = NO \\ 1\_if\_BLANK\_SUBTRACTION = YES \end{Bmatrix}$$

NOTE: $\delta_1 = \delta_2 = 1$ IS NOT CURRENTLY ALLOWED

Beta Count Rate in the ATB Mode:

Since the ATB mode matches one α only count to a corresponding $[\alpha + \beta]$ count, and the one (and only) blank is counted during the same ATB count, then the count times are the same, and we may write:

$$T_{[\alpha+\beta]} = T_{\alpha} = T_{[\alpha+\beta]\_blank} = T_{\alpha\_blank} = T$$

$$R_{\beta} = (R_{[\alpha+\beta]} - \delta_2 \cdot R_{[\alpha+\beta]\_blank} - \delta_1 \cdot [1-\delta_2] \cdot R_{B\_[\alpha+\beta]}) - (R_{\alpha} - \delta_2 \cdot R_{\alpha\_blank} - \delta_1 \cdot [1-\delta_2] \cdot R_{B\_\alpha})$$

$$R_{\beta} = R_{[\alpha+\beta]} - \delta_2 \cdot R_{[\alpha+\beta]\_blank} - \delta_1 \cdot [1-\delta_2] \cdot R_{B\_[\alpha+\beta]} - R_{\alpha} + \delta_2 \cdot R_{\alpha\_blank} + \delta_1 \cdot [1-\delta_2] \cdot R_{B\_\alpha}$$

$$R_{\beta} = R_{[\alpha+\beta]} - R_{\alpha} - \delta_2 \cdot (R_{[\alpha+\beta]\_blank} - R_{\alpha\_blank}) - \delta_1 \cdot [1-\delta_2] \cdot (R_{B\_[\alpha+\beta]} - R_{B\_\alpha})$$

OR

$$R_{\beta} = R_{[\alpha+\beta]} - R_{\alpha} - \delta_2 \cdot (R_{[\alpha+\beta]\_blank} - R_{\alpha\_blank}) - \delta_1 \cdot [1-\delta_2] \cdot (R_{B\_\beta})$$

where,

$R_{[\alpha+\beta]} =$ the gross ($\alpha + \beta$) count rate obtained during the [$\alpha + \beta$] mode of the ATB count.

$R_{\alpha} =$ the gross (α) ONLY count rate obtained during the [α] ONLY mode of the ATB count.

$R_{[\alpha+\beta]\_blank} =$ the gross ($\alpha + \beta$) count rate of the (one and only one) designated blank obtained during the [$\alpha + \beta$] count of the blank during the [$\alpha + \beta$] mode of the ATB count.

$R_{\alpha\_blank} =$ the gross (α) ONLY count rate of the (one and only one) designated blank obtained during the α ONLY count of the blank during the α ONLY mode of the ATB count.

$R_{B\_[\alpha+\beta]} =$ the [$\alpha + \beta$] (system) BACKGROUND count rate for the [$\alpha + \beta$] mode of the ATB count mode.

$R_{B\_\alpha} =$ the α (system) BACKGROUND count rate for the α ONLY mode of the ATB count mode.

$R_{B\_\beta} =$ the **derived** β (system) BACKGROUND count rate for the ATB count mode.

$$\delta_1 = \begin{Bmatrix} 0\_if\_BACKGROUND\_SUBTRACTION = NO \\ 1\_if\_BACKGROUND\_SUBTRACTION = YES \end{Bmatrix}$$

$$\delta_2 = \begin{Bmatrix} 0\_if\_BLANK\_SUBTRACTION = NO \\ 1\_if\_BLANK\_SUBTRACTION = YES \end{Bmatrix}$$

Note: $\delta_1 = \delta_2 = 1$ IS NOT CURRENTLY ALLOWED

Uncertainties:

Once again, noting that

$$T_{[\alpha+\beta]} = T_{\alpha} = T_{[\alpha+\beta]\_blank} = T_{\alpha\_blank} = T$$

we may write

$$\sigma_{R_\beta} = \sqrt{\left(\frac{R_{[\alpha+\beta]}}{T}\right) + \left(\frac{R_\alpha}{T}\right) + \delta_2^2 \cdot \left[\left(\frac{R_{[\alpha+\beta]\_blank}}{T}\right) + \left(\frac{R_{\alpha\_blank}}{T}\right)\right] + \delta_1^2 \cdot [1-\delta_2]^2 \cdot (\sigma_{B\_[\alpha+\beta]}^2 + \sigma_{B\_\alpha}^2)}$$

where,

$R_{[\alpha+\beta]} =$ the gross ($\alpha+\beta$) count rate obtained during the [$\alpha+\beta$] mode of the ATB count.

$R_\alpha =$ the gross $\alpha-$ONLY count rate obtained during the [α] ONLY mode of the ATB count.

$R_{[\alpha+\beta]\_blank} =$ the gross ($\alpha+\beta$) count rate of the (one and only one) designated blank obtained during the [$\alpha+\beta$] count of the blank during the [$\alpha+\beta$] mode of the ATB count.

$R_{\alpha\_blank} =$ the gross $\alpha-$ONLY count rate of the (one and only one) designated blank obtained during the [α]ONLY count of the blank during the [α]ONLY mode of the ATB count.

$R_{B\_[\alpha+\beta]} =$ the ($\alpha+\beta$) (system) BACKGROUND count rate for the [$\alpha+\beta$] mode of the ATB count mode.

$R_{B\_\alpha} =$ the $\alpha-$ONLY (system) BACKGROUND count rate for the [α]ONLY mode of the ATB count mode.

$\sigma_{B\_[\alpha+\beta]} =$ the **uncertainty** in the ($\alpha+\beta$) (system) BACKGROUND count rate for the [$\alpha+\beta$] mode of the ATB count mode.

$$= \sqrt{\frac{R_{B\_[\alpha+\beta]}}{T_B}}$$ for the case in which the background was determined from a single measurement

$$= \sqrt{\frac{\sum_{i=1}^{N}\left(R_{B\_[\alpha+\beta]\_i} - R_{B\_[\alpha+\beta]}\right)^2}{(N-1)}}$$ if the background was determined from a set of N measurements.

$\sigma_{B\_\alpha} =$ the **uncertainty** in the α (system) BACKGROUND count rate for the α ONLY mode of the ATB count mode.

$$= \sqrt{\frac{R_{B\_\alpha}}{T_B}}$$ for the case in which the background was determined from a single measurement

$$= \sqrt{\frac{\sum_{i=1}^{N}\left(R_{B\_\alpha\_i} - R_{B\_\alpha}\right)^2}{(N-1)}}$$

if the background was determined from a set of N measurements.

$$\delta_1 = \begin{cases} 0\_if\_BACKGROUND\_SUBTRACTION = NO \\ 1\_if\_BACKGROUND\_SUBTRACTION = YES \end{cases}$$

$$\delta_2 = \begin{cases} 0\_if\_BLANK\_SUBTRACTION = NO \\ 1\_if\_BLANK\_SUBTRACTION = YES \end{cases}$$

NOTE: $\delta_1 = \delta_2 = 1$ IS NOT CURRENTLY ALLOWED

NOTE: Care should be taken in selecting background subtraction versus method blank subtraction during the alpha then beta mode. Unlike the method blank protocol (in modes other than ATB) in which the result of subtracting the gross blank from the gross sample produces the same result as subtracting the net blank from the net sample, in the ATB mode, subtracting the gross α only count from the gross ($\alpha + \beta$) count does not produce the same result as subtracting the net α only count from the net ($\alpha + \beta$) count. The difference is the derived β background. While the δ_1 parameter ensures that system background is consistently applied to the α only and ($\alpha + \beta$) components of the above equations for the ATB mode, it needs to be noted that the system backgrounds for these modes are different. (The difference is the derived β background.) In fact, the δ_1 parameter multiplies what one could call the "derived beta background". Thus turning background subtraction ON versus OFF determines whether the "derived beta background" is subtracted or not – consistent with the definition of net count versus gross count. In other words, (derived gross β – β background) and (net [$\alpha + \beta$] – net [α only]) produce the same net β only result (760 in the table below).

In a similar fashion, when one tries to take into account a method blank, consistent results will be obtained provided one is consistent in matching gross with gross and net with net measurements.

The following table helps to demonstrate these concepts:

TABLE

NOTE: In the context of this Table, the following definitions will apply:
TOTAL = sample contribution + blank contribution
GROSS = including system background
NET = the system background has been subtracted.

MODE:	(α ONLY)	($\alpha + \beta$)	DERIVED β TOTAL GROSS β	β SYS BACK	BLANK GROSS β	BLANK NET β
Observed Backgrounds:	**10**	**50**				
Derived Background:				**40**		
Observed Blank:	**30**	400				
Derived Gross Blank:					**370**	
Derived Net Blank:	**20**	**350**				**330**
Observed Count:	**100**	**900**				
Derived Gross Count:			**800**	800	800	
Derived Total Net Beta Count:						**760**
$\delta_1 = 0, \delta_2 = 0$ **: Gross Total SAMPLE:**	**100**	**900**	**800**[1]			
$\delta_1 = 1, \delta_2 = 0$ **: Gross Total Samp - SYS BACK:**	**90**	**850**		**760**[2]		
$\delta_1 = 0, \delta_2 = 1$ **: Gross Total Samp - Gross Blank:**	**70**	**500**			**430**[3]	
$\delta_1 = 1, \delta_2 = 1$ **:Net Total Samp – Net Blank:**	**(90-20)**	**(850-350)**				**430**[4]

NOTE 1: The TOTAL gross beta counts. These are due to contributions from:

the system background = 40
The beta contribution from the method = 330
And the beta contribution from the sample itself = 430
800

NOTE 2: The TOTAL net beta counts. These are due to contributions from:

The beta contribution from the method = 330
And the beta contribution from the sample itself = 430
760

NOTE 3: The NET SAMPLE beta counts. These are due to contributions from:

the NET beta contribution from the sample itself = 430
430

While this is obtained by subtracting the GROSS blank count rate from the GROSS sample count rate, it still produces the NET beta count rate because the system background contribution is common to both the sample and blank, and is thus subtracted out:

The GROSS sample count rate =	800	(40+330+430)
the GROSS blank count rate =	370	(40+330)
	430	

NOTE 4: The NET SAMPLE beta counts. These are due to contributions from:

the NET beta contribution from the sample itself =	430
	430

This result is obtained by subtracting the NET blank count rate from the NET sample count rate. It produces the same NET beta count rate as demonstrated in NOTE 3:

The NET sample count rate =	760	(330+430)
the NET blank count rate =	330	(330)
	430	

These results follow from the associative property of addition and subtraction.

5. Efficiency and Spillover Calibrations without Mass Attenuation

5.1 Efficiency:

For: N≥1

$$\varepsilon = \frac{1}{N} \cdot \sum_{i=1}^{N} \varepsilon_i = \frac{1}{N} \cdot \sum_{i=1}^{N} \frac{R_{i\_calc'd}}{S}$$

where,

$\varepsilon_i = \dfrac{R_{i\_calc'd}}{S} =$ the efficiency determined from the ith observation.

$R_{i\_calc'd} =$ the "calculated" count rate (net or gross as determined by the analysis profile) for the particle $(\alpha$ or $\beta)$ of interest during the ith observation.

$S =$ the emission rate of the calibration standard for the particle of interest.

$= S_0 \cdot e^{-\lambda \cdot \Delta T}$

$S_0 =$ the emission rate of the calibration standard as of the certificate date.

$\lambda = \dfrac{\ln(2)}{T_{1/2}}$

$\Delta T =$ Elapsed time between Calibration source certificate date/time and the Count Acquisition Date/Time.

Uncertainty:

For $N = 1$:

$$\frac{\sigma_{\varepsilon_1}}{\varepsilon_1} = \sqrt{\left(\frac{\sigma_{R_{1\_calc'd}}}{R_{1\_calc'd}}\right)^2 + \left(\frac{\sigma_S}{S}\right)^2}$$

$\sigma_{R_1} =$ Uncertainty in the "calculated" count rate

From the count rate determination as stored in the database.

$\sigma_S =$ Uncertainty in the emission rate of the calibration standard.

For $N > 1$:

$$\sigma_{\bar{\varepsilon}}^2 = \frac{1}{(N-1)} \cdot \sum_{i=1}^{N} (\varepsilon_i - \varepsilon)^2$$

5.2 Spillover – Simultaneous Mode Only:

Spilldown – Determined during alpha efficiency in Simultaneous Mode:

For a single measurement:

$$\chi_{1\_\alpha->\beta} = \frac{R_{1\_\beta\_calc'd}}{R_{1\_\alpha\_calc'd}}$$

and

$$\frac{\sigma_{\chi_{1\_\alpha->\beta}}}{\chi_{1\_\alpha->\beta}} = \sqrt{\left(\frac{\sigma_{R_{1\_\beta\_calc'd}}}{R_{1\_\beta\_calc'd}}\right)^2 + \left(\frac{\sigma_{R_{1\_\alpha\_calc'd}}}{R_{1\_\alpha\_calc'd}}\right)^2}$$

or restated as

$$\sigma_{\chi_{1\_\alpha->\beta}} = \chi_{1\_\alpha->\beta} \cdot \sqrt{\left(\frac{\sigma_{R_{1\_\beta\_calc'd}}}{R_{1\_\beta\_calc'd}}\right)^2 + \left(\frac{\sigma_{R_{1\_\alpha\_calc'd}}}{R_{1\_\alpha\_calc'd}}\right)^2}$$

where,

$R_{1\_\beta\_calc'd}$ = the "calculated" beta count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the **one** observation.

$R_{1\_\alpha\_calc'd}$ = the "calculated" alpha count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the **one** observation.

$\sigma_{R_{1\_\alpha\_calc'd}}$ = the uncertainty in the "calculated" alpha count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the **one** observation.

$\sigma_{R_{1\_\beta\_calc'd}}$ = the uncertainty in the "calculated" beta count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the **one** observation.

For a set of *N* measurements:

$$\chi_{\alpha->\beta} = \frac{1}{N} \cdot \sum_{i=1}^{N} \chi_{i\_\alpha->\beta}$$

where,

$$\chi_{i\_\alpha->\beta} = \frac{R_{i\_\beta\_calc'd}}{R_{i\_\alpha\_calc'd}}$$

$R_{i\_\beta\_calc'd}$ = the "calculated" beta count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the ith observation.

$R_{i\_\alpha\_calc'd}$ = the "calculated" alpha count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the ith observation.

and

$$\sigma^2_{\chi_{\alpha->\beta}} = \frac{1}{(N-1)} \cdot \sum_{i=1}^{N} \left(\chi_{i\_\alpha->\beta} - \chi_{\alpha->\beta}\right)^2$$

Spillup – Determined during beta efficiency in Simultaneous Mode:

For a single measurement:

$$\chi_{1\_\beta->\alpha} = \frac{R_{1\_\alpha\_calc'd}}{R_{1\_\beta\_calc'd}}$$

and

$$\frac{\sigma_{\chi_{1\_\beta->\alpha}}}{\chi_{1\_\beta->\alpha}} = \sqrt{\left(\frac{\sigma_{R_{1\_\beta\_calc'd}}}{R_{1\_\beta\_calc'd}}\right)^2 + \left(\frac{\sigma_{R_{1\_\alpha\_calc'd}}}{R_{1\_\alpha\_calc'd}}\right)^2}$$

or restated as

$$\sigma_{\chi_{1\_\beta->\alpha}} = \chi_{1\_\beta->\alpha} \cdot \sqrt{\left(\frac{\sigma_{R_{1\_\beta\_calc'd}}}{R_{1\_\beta\_calc'd}}\right)^2 + \left(\frac{\sigma_{R_{1\_\alpha\_calc'd}}}{R_{1\_\alpha\_calc'd}}\right)^2}$$

where,

$R_{1\_\alpha\_calc'd}$ = the “calculated” alpha count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the **one** observation.

$R_{1\_\beta\_calc'd}$ = the “calculated” beta count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the **one** observation.

$\sigma_{R_{1\_\alpha\_calc'd}}$ = the uncertainty in the “calculated” alpha count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the **one** observation.

$\sigma_{R_{1\_\beta\_calc'd}}$ = the uncertainty in the “calculated” beta count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the **one** observation.

For a set of *N* measurements:

$$\chi_{\beta->\alpha} = \frac{1}{N} \cdot \sum_{i=1}^{N} \chi_{i\_\beta->\alpha}$$

where,

$$\chi_{i\_\beta->\alpha} = \frac{R_{i\_\alpha\_calc'd}}{R_{i\_\beta\_calc'd}}$$

$R_{i\_\alpha\_calc'd}$ = the “calculated” alpha count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the ith observation.

$R_{i\_\beta\_calc'd}$ = the “calculated” beta count rate (net or gross as determined by the analysis profile) in the simultaneous count mode for the ith observation.

and

$$\sigma^2_{\chi_{\beta->\alpha}} = \frac{1}{(N-1)} \cdot \sum_{i=1}^{N} \left(\chi_{i\_\beta->\alpha} - \chi_{\beta->\alpha}\right)^2$$

6. Efficiency and Spillover Calibrations with Mass Attenuation

6.1 Efficiency Calibrations with Mass Attenuation

The efficiency for samples of non-zero mass is modeled as a function of mass. Four models are available:

* Linear: $\varepsilon(m) = C_0 + C_1 \cdot m$
* Exponential: $\varepsilon(m) = C_0 \cdot e^{-C_1 \cdot m}$
* Inverse Linear: $\varepsilon(m) = [C_0 + C_1 \cdot m]^{-1}$
* Inverse Quadratic: $\varepsilon(m) = \left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^{-1}$

The coefficients of the equation (for the selected model) are determined from a **weighted** least squares fit to a set of paired mass-efficiency observations: [(m_1, ε_1);(m_2, ε_2);….;(m_N, ε_N)]. These coefficients, along with their uncertainties, are stored so that the efficiency, and its uncertainty, for a sample of any (attenuating) mass can be calculated and used for an activity determination.

Linear

The Fitted Efficiency

The coefficients of the linear solution:

$$\varepsilon(m) = C_0 + C_1 \cdot m$$

are determined by solving for the vector $\bar{b}$ in the following equations:

$$M \cdot \bar{b} = \bar{V}$$

where,

$$M_{JK} = \sum_{i=1}^{N} w_i \cdot m_i^{J-1} \cdot m_i^{K-1}$$

$$\bar{b} = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix}$$

$$V_K = \sum_{i=1}^{N} w_i \cdot \varepsilon_i \cdot m_i^{K-1}$$

and

$$w_i = \frac{1}{\sigma_{\varepsilon_i}^2}$$

The coefficients are then given by

$$\bar{b} = M^{-1} \cdot M \cdot \bar{b} = M^{-1} \cdot \bar{V}$$

and the uncertainties in the coefficients are given by

$$\sigma_{C_0}^2 = M_{11}^{-1}$$

$$\sigma_{C_1}^2 = M_{22}^{-1}$$

The reduced χ^2 of the final fit, Z, is given by:

$$Z = \frac{\chi^2}{N-2}$$

where,

$$\chi^2 = \sum_{i=1}^{N} \frac{[\varepsilon_i - \varepsilon(m_i)]^2}{\sigma_{\varepsilon_i}^2}$$

For good fits, $Z \to 1$.

The Uncertainty in the Fitted Efficiency

The uncertainty in the calculated efficiency is determined as follows:

Given that the efficiency is calculated from the following equation:

$$\varepsilon(m) = C_0 + C_1 \cdot m$$

we can write

$$(d\varepsilon)^2 = \left[\frac{\partial\varepsilon}{\partial C_0}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{\partial\varepsilon}{\partial C_1}\right]^2 \cdot \sigma_{C_1}^2 + \left[\frac{\partial\varepsilon}{\partial m}\right]^2 \cdot \sigma_m^2$$

$$= [1]^2 \cdot \sigma_{C_0}^2 + [m]^2 \cdot \sigma_{C_1}^2 + [C_1]^2 \cdot \sigma_m^2$$

$$= \sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + C_1^2 \cdot \sigma_m^2$$

Substituting

$$\sigma_{C_0}^2 = M_{11}^{-1}$$

$$\sigma_{C_1}^2 = M_{22}^{-1}$$

we can write

$$d\varepsilon^2 = M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2$$

or

$$d\varepsilon = \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

Exponential

The Fitted Efficiency

The coefficients of the exponential solution:

$$\varepsilon(m) = C_0 \cdot e^{-C_1 \cdot m}$$

are determined by first linearizing the equation:

$$y = \ln[\varepsilon(m)] = \ln(C_0) - C_1 \cdot m = A_0 + A_1 \cdot m$$

and then solving for A_0 and A_1 by solving for the vector $\bar{b}$ in the equation

$$M \cdot \bar{b} = \bar{V}$$

in which,

$$M_{JK} = \sum_{i=1}^{N} w_i \cdot m_i^{J-1} \cdot m_i^{K-1}$$

$$\bar{b} = \begin{bmatrix} A_0 \\ A_1 \end{bmatrix}$$

$$V_K = \sum_{i=1}^{N} w_i \cdot \ln(\varepsilon_i) \cdot m_i^{K-1}$$

and

$$w_i = \frac{1}{\sigma_y^2}$$

Since $y = \ln(\varepsilon)$

and $(dy)^2 = \left(\frac{1}{\varepsilon}\right)^2 \cdot (d\varepsilon)^2$

the variance of y is given by

$$\sigma_y^2 = \left(\frac{1}{\varepsilon}\right)^2 \cdot \sigma_\varepsilon^2$$

so that the weighting factor, w_i, is given by

$$w_i = \frac{\varepsilon^2}{\sigma_\varepsilon^2}$$

Then, noting that

$$A_0 = \ln(C_0)$$
$$A_1 = -C_1$$

we may now write:

$$C_0 = e^{A_0}$$
$$C_1 = -A_1$$

The coefficients, A_0 and A_1, are given by

$$\bar{b} = \begin{bmatrix} A_0 \\ A_1 \end{bmatrix} = M^{-1} \cdot M \cdot \bar{b} = M^{-1} \cdot \bar{V}$$

and, as before, the uncertainties in the coefficients A_0 and A_1 are given by

$$\sigma_{A_0}^2 = M_{11}^{-1}$$
$$\sigma_{A_1}^2 = M_{22}^{-1}$$

Noting the uncertainties in the coefficients C_0 and C_1, are given by

$$\sigma_{C_0} = e^{A_0} \cdot \sigma_{A_0}$$
$$\sigma_{C_1} = \sigma_{A_1}$$

and substituting for σ_{A_0} and σ_{A_1} in the above equations, we obtain:

$$\sigma_{C_0} = e^{A_0} \cdot \sigma_{A_0} = e^{A_0} \cdot \sqrt{M_{11}^{-1}} = C_0 \cdot \sqrt{M_{11}^{-1}}$$
$$\sigma_{C_1} = \sigma_{A_1} = \sqrt{M_{22}^{-1}}$$

As before, the reduced χ^2 of the final fit is given by:

$$Z = \frac{\chi^2}{N-2}$$

where,

$$\chi^2 = \sum_{i=1}^{N} \frac{\left[\varepsilon_i - \varepsilon(m_i)\right]^2}{\sigma_{\varepsilon_i}^2}$$

Once again, for good fits, $Z \to 1$.

The Uncertainty in the Fitted Efficiency

The uncertainty in the calculated efficiency is determined as follows:

Given that the efficiency is calculated from the following equation:

$$\varepsilon(m) = C_0 \cdot e^{-C_1 \cdot m}$$

we can write

$$(d\varepsilon)^2 = \left[\frac{\partial \varepsilon}{\partial C_0}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{\partial \varepsilon}{\partial C_1}\right]^2 \cdot \sigma_{C_1}^2 + \left[\frac{\partial \varepsilon}{\partial m}\right]^2 \cdot \sigma_m^2$$

$$= \left[e^{-C_1 \cdot m}\right]^2 \cdot \sigma_{C_0}^2 + \left[C_0 \cdot e^{-C_1 \cdot m} \cdot (-m)\right]^2 \cdot \sigma_{C_1}^2 + \left[C_0 \cdot e^{-C_1 \cdot m} \cdot (-C_1)\right]^2 \cdot \sigma_m^2$$

$$= \left[e^{-C_1 \cdot m}\right]^2 \bullet \left[\sigma_{C_0}^2 + C_0^2 \cdot m^2 \cdot \sigma_{C_1}^2 + C_0^2 \cdot C_1^2 \cdot \sigma_m^2\right]$$

$$= \left[C_0 \cdot e^{-C_1 \cdot m}\right]^2 \bullet \left[\frac{\sigma_{C_0}^2}{C_0^2} + m^2 \cdot \sigma_{C_1}^2 + C_1^2 \cdot \sigma_m^2\right]$$

Substituting

$$\varepsilon(m) = C_0 \cdot e^{-C_1 \cdot m}$$

$$\sigma_{C_0} = e^{A_0} \cdot \sigma_{A_0} = e^{A_0} \cdot \sqrt{M_{11}^{-1}} = C_0 \cdot \sqrt{M_{11}^{-1}}$$

$$\sigma_{C_1} = \sigma_{A_1} = \sqrt{M_{22}^{-1}}$$

we can write

$$(d\varepsilon)^2 = \varepsilon^2 \bullet \left[M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2\right]$$

or

$$d\varepsilon = \varepsilon \bullet \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

or

$$\frac{d\varepsilon}{\varepsilon} = \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

Inverse Linear

The Fitted Efficiency

The coefficients of the inverse linear solution:

$$\varepsilon(m) = [C_0 + C_1 \cdot m]^{-1}$$

are determined by first linearizing the equation:

$$y = \frac{1}{\varepsilon(m)} = C_0 + C_1 \cdot m$$

and then solving for C_0 and C_1 by solving for the vector $\bar{b}$ in the equation

$$M \cdot \bar{b} = \bar{V}$$

in which,

$$M_{JK} = \sum_{i=1}^{N} w_i \cdot m_i^{J-1} \cdot m_i^{K-1}$$

$$\bar{b} = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix}$$

$$V_K = \sum_{i=1}^{N} w_i \cdot \frac{1}{\varepsilon_i} \cdot m_i^{K-1}$$

and

$$w_i = \frac{1}{\sigma_y^2}$$

Since $y = \dfrac{1}{\varepsilon}$

and $(dy)^2 = \left(\dfrac{-1}{\varepsilon^2}\right)^2 \cdot (d\varepsilon)^2$

the variance of y is given by

$$\sigma_y^2 = \frac{1}{\varepsilon^4} \cdot \sigma_\varepsilon^2$$

so that the weighting factor, w_i, is given by

$$w_i = \frac{\varepsilon^4}{\sigma_\varepsilon^2}$$

The coefficients, C_0 and C_1, are given by

$$\bar{b} = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = M^{-1} \cdot M \cdot \bar{b} = M^{-1} \cdot \bar{V}$$

The uncertainties in the coefficients C_0 and C_1 are given by

$$\sigma_{C_0}^2 = M_{11}^{-1}$$
$$\sigma_{C_1}^2 = M_{22}^{-1}$$

As before, the reduced χ^2 of the final fit is given by:

$$Z = \frac{\chi^2}{N-2}$$

where,

$$\chi^2 = \sum_{i=1}^{N} \frac{[\varepsilon_i - \varepsilon(m_i)]^2}{\sigma_{\varepsilon_i}^2}$$

Once again, for good fits, $Z \to 1$.

The Uncertainty in the Fitted Efficiency

The uncertainty in the calculated efficiency is determined as follows:

Given that the efficiency is calculated from the following equation:

$$\varepsilon(m) = [C_0 + C_1 \cdot m]^{-1}$$

we can write

$$(d\varepsilon)^2 = \left[\frac{\partial \varepsilon}{\partial C_0}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{\partial \varepsilon}{\partial C_1}\right]^2 \cdot \sigma_{C_1}^2 + \left[\frac{\partial \varepsilon}{\partial m}\right]^2 \cdot \sigma_m^2$$

$$= \left[\frac{-1}{[C_0 + C_1 \cdot m]^2}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{-1}{[C_0 + C_1 \cdot m]^2}\right]^2 \cdot [m]^2 \cdot \sigma_{C_1}^2 + \left[\frac{-1}{[C_0 + C_1 \cdot m]^2}\right]^2 \cdot [C_1]^2 \cdot \sigma_m^2$$

$$= \left[\frac{-1}{[C_0 + C_1 \cdot m]^2}\right]^2 \bullet \left[\sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + C_1^2 \cdot \sigma_m^2\right]$$

Substituting

$$\varepsilon(m) = [C_0 + C_1 \cdot m]^{-1}$$

We obtain

$$(d\varepsilon)^2 = \varepsilon^4 \bullet \left[\sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + C_1^2 \cdot \sigma_m^2\right]$$

Now substituting

$$\sigma_{C_0}^2 = M_{11}^{-1}$$

$$\sigma_{C_1}^2 = M_{22}^{-1}$$

We obtain

$$(d\varepsilon)^2 = \varepsilon^4 \bullet \left[M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2\right]$$

or

$$d\varepsilon = \varepsilon^2 \bullet \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

or

$$\frac{d\varepsilon}{\varepsilon} = \varepsilon \bullet \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

Inverse Quadratic

The Fitted Efficiency

The coefficients of the inverse quadratic solution:

$$\varepsilon(m) = \left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^{-1}$$

are determined by first linearizing the equation:

$$y = \frac{1}{\varepsilon(m)} = C_0 + C_1 \cdot m + C_2 \cdot m^2$$

and then solving for C_0, C_1, and C_2 by solving for the vector $\overline{b}$ in the equation

$$M \cdot \overline{b} = \overline{V}$$

in which,

$$M_{JK} = \sum_{i=1}^{N} w_i \cdot m_i^{J-1} \cdot m_i^{K-1}$$

$$\overline{b} = \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix}$$

$$V_K = \sum_{i=1}^{N} w_i \cdot \frac{1}{\varepsilon_i} \cdot m_i^{K-1}$$

and

$$w_i = \frac{1}{\sigma_y^2}$$

Since $y = \dfrac{1}{\varepsilon}$

and $(dy)^2 = \left(\dfrac{-1}{\varepsilon^2}\right)^2 \cdot (d\varepsilon)^2$

the variance of y is given by

$$\sigma_y^2 = \frac{1}{\varepsilon^4} \cdot \sigma_\varepsilon^2$$

so that the weighting factor, w_i, is given by

$$w_i = \frac{\varepsilon^4}{\sigma_\varepsilon^2}$$

The coefficients, C_0, C_1, and C_2, are given by

$$\bar{b} = \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix} = M^{-1} \cdot M \cdot \bar{b} = M^{-1} \cdot \bar{V}$$

The uncertainties in the coefficients C_0, C_1, and C_2 are given by

$$\sigma_{C_0}^2 = M_{11}^{-1}$$
$$\sigma_{C_1}^2 = M_{22}^{-1}$$
$$\sigma_{C_2}^2 = M_{33}^{-1}$$

With three coefficients, the reduced χ^2 of the final fit is given by:

$$Z = \frac{\chi^2}{N-3}$$

where,

$$\chi^2 = \sum_{i=1}^{N} \frac{[\varepsilon_i - \varepsilon(m_i)]^2}{\sigma_{\varepsilon_i}^2}$$

Once again, for good fits, $Z \to 1$.

The Uncertainty in the Fitted Efficiency

The uncertainty in the calculated efficiency is determined as follows:

Given that the efficiency is calculated from the following equation:

$$\varepsilon(m) = \left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^{-1}$$

we can write

$$(d\varepsilon)^2 = \left[\frac{\partial\varepsilon}{\partial C_0}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{\partial\varepsilon}{\partial C_1}\right]^2 \cdot \sigma_{C_1}^2 + \left[\frac{\partial\varepsilon}{\partial C_2}\right]^2 \cdot \sigma_{C_2}^2 + \left[\frac{\partial\varepsilon}{\partial m}\right]^2 \cdot \sigma_m^2$$

$$= \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \cdot \sigma_{C_0}^2$$

$$+ \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \cdot (m)^2 \cdot \sigma_{C_1}^2$$

$$+ \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \cdot (m^2)^2 \cdot \sigma_{C_2}^2$$

$$+ \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \cdot (2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2$$

$$(d\varepsilon)^2 = \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \bullet \left[\sigma_{C_0}^2 + (m)^2 \cdot \sigma_{C_1}^2 + (m^2)^2 \cdot \sigma_{C_2}^2 + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2\right]$$

$$= \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \bullet \left[\sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + m^4 \cdot \sigma_{C_2}^2 + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2\right]$$

Substituting $\varepsilon(m) = \left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^{-1}$

We obtain

$$(d\varepsilon)^2 = \varepsilon^4 \bullet \left[\sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + m^4 \cdot \sigma_{C_2}^2 + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2\right]$$

Now substituting

$$\sigma_{C_0}^2 = M_{11}^{-1}$$
$$\sigma_{C_1}^2 = M_{22}^{-1}$$
$$\sigma_{C_2}^2 = M_{33}^{-1}$$

We obtain

$$(d\varepsilon)^2 = \varepsilon^4 \bullet \left[M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + m^4 \cdot M_{33}^{-1} + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2\right]$$

or

$$d\varepsilon = \varepsilon^2 \bullet \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + m^4 \cdot M_{33}^{-1} + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2}$$

or

$$\frac{d\varepsilon}{\varepsilon} = \varepsilon \bullet \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + m^4 \cdot M_{33}^{-1} + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2}$$

6.2 Spillover Calibrations (Simultaneous Mode Only) with Mass Attenuation

Note: Traditionally the symbol χ has been used to represent spillover; however, in this section we will also be working with the Chi-Squared (χ^2) value of the least squares fit. Therefore, to avoid confusion between the spillover and the Chi-Squared value of the fit, the symbol "S" will be used in this section to represent spillover.

The spillover for samples of non-zero mass is modeled as a function of mass. Four models are available:

* Linear: $S(m) = C_0 + C_1 \cdot m$
* Exponential: $S(m) = C_0 \cdot e^{-C_1 \cdot m}$
* Inverse Linear: $S(m) = [C_0 + C_1 \cdot m]^{-1}$
* Inverse Quadratic: $S(m) = [C_0 + C_1 \cdot m + C_2 \cdot m^2]^{-1}$

The coefficients of the equation (for the selected model) are determined from a **weighted** least squares fit to a set of paired mass-spillover observations: [(m_1, S_1);(m_2, S_2);….;(m_N, S_N)]. These coefficients, along with their uncertainties, are stored so that the spillover, and its uncertainty, for a sample of any (attenuating) mass can be calculated and used for an activity determination.

Linear

The Fitted Spillover

The coefficients of the linear solution:

$$S(m) = C_0 + C_1 \cdot m$$

are determined by solving for the vector $\bar{b}$ in the following equations:

$$M \cdot \bar{b} = \bar{V}$$

where,

$$M_{JK} = \sum_{i=1}^{N} w_i \cdot m_i^{J-1} \cdot m_i^{K-1}$$

$$\bar{b} = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix}$$

$$V_K = \sum_{i=1}^{N} w_i \cdot \chi_i \cdot m_i^{K-1}$$

and

$$w_i = \frac{1}{\sigma_{\chi_i}^2}$$

The coefficients are then given by

$$\bar{b} = M^{-1} \cdot M \cdot \bar{b} = M^{-1} \cdot \bar{V}$$

and the uncertainties in the coefficients are given by

$$\sigma_{C_0}^2 = M_{11}^{-1}$$

$$\sigma_{C_1}^2 = M_{22}^{-1}$$

The reduced χ^2 of the final fit is given by:

$$Z = \frac{\chi^2}{N-2}$$

where,

$$\chi^2 = \sum_{i=1}^{N} \frac{\left[S_i - S(m_i)\right]^2}{\sigma_{S_i}^2}$$

For good fits, $Z \rightarrow 1$.

The Uncertainty in the Fitted Spillover

The uncertainty in the calculated spillover is determined as follows:

Given that the spillover is calculated from the following equation:

$$S(m) = C_0 + C_1 \cdot m$$

we can write

$$(dS)^2 = \left[\frac{\partial S}{\partial C_0}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{\partial S}{\partial C_1}\right]^2 \cdot \sigma_{C_1}^2 + \left[\frac{\partial S}{\partial m}\right]^2 \cdot \sigma_m^2$$

$$= [1]^2 \cdot \sigma_{C_0}^2 + [m]^2 \cdot \sigma_{C_1}^2 + [C_1]^2 \cdot \sigma_m^2$$

$$= \sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + C_1^2 \cdot \sigma_m^2$$

Substituting

$$\sigma_{C_0}^2 = M_{11}^{-1}$$

$$\sigma_{C_1}^2 = M_{22}^{-1}$$

we can write

$$dS^2 = M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2$$

or

$$dS = \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

Exponential

The Fitted Spillover

The coefficients of the exponential solution:

$$S(m) = C_0 \cdot e^{-C_1 \cdot m}$$

are determined by first linearizing the equation:

$$y = \ln[S(m)] = \ln(C_0) - C_1 \cdot m = A_0 + A_1 \cdot m$$

and then solving for A_0 and A_1 by solving for the vector $\bar{b}$ in the equation

$$M \cdot \bar{b} = \bar{V}$$

in which,

$$M_{JK} = \sum_{i=1}^{N} w_i \cdot m_i^{J-1} \cdot m_i^{K-1}$$

$$\bar{b} = \begin{bmatrix} A_0 \\ A_1 \end{bmatrix}$$

$$V_K = \sum_{i=1}^{N} w_i \cdot \ln(S_i) \cdot m_i^{K-1}$$

and

$$w_i = \frac{1}{\sigma_y^2}$$

Since $$y = \ln(S)$$

and $$(dy)^2 = \left(\frac{1}{S}\right)^2 \cdot (dS)^2$$

the variance of y is given by

$$\sigma_y^2 = \left(\frac{1}{S}\right)^2 \cdot \sigma_S^2$$

so that the weighting factor, w_i, is given by

$$w_i = \frac{S^2}{\sigma_S^2}$$

Then, noting that

$$A_0 = \ln(C_0)$$
$$A_1 = -C_1$$

we may now write:

$$C_0 = e^{A_0}$$
$$C_1 = -A_1$$

The coefficients, A_0 and A_1, are given by

$$\bar{b} = \begin{bmatrix} A_0 \\ A_1 \end{bmatrix} = M^{-1} \cdot M \cdot \bar{b} = M^{-1} \cdot \bar{V}$$

and, as before, the uncertainties in the coefficients A_0 and A_1 are given by

$$\sigma_{A_0}^2 = M_{11}^{-1}$$
$$\sigma_{A_1}^2 = M_{22}^{-1}$$

Noting the uncertainties in the coefficients C_0 and C_1, are given by

$$\sigma_{C_0} = e^{A_0} \cdot \sigma_{A_0}$$
$$\sigma_{C_1} = \sigma_{A_1}$$

and substituting for σ_{A_0} and σ_{A_1} in the above equations, we obtain:

$$\sigma_{C_0} = e^{A_0} \cdot \sigma_{A_0} = e^{A_0} \cdot \sqrt{M_{11}^{-1}} = C_0 \cdot \sqrt{M_{11}^{-1}}$$
$$\sigma_{C_1} = \sigma_{A_1} = \sqrt{M_{22}^{-1}}$$

As before, the reduced χ^2 of the final fit is given by:

$$Z = \frac{\chi^2}{N-2}$$

where,

$$\chi^2 = \sum_{i=1}^{N} \frac{\left[S_i - S(m_i)\right]^2}{\sigma_{S_i}^2}$$

Once again, for good fits, $Z \to 1$.

The Uncertainty in the Fitted Spillover

The uncertainty in the calculated spillover is determined as follows:

Given that the spillover is calculated from the following equation:

$$S(m) = C_0 \cdot e^{-C_1 \cdot m}$$

we can write

$$(dS)^2 = \left[\frac{\partial S}{\partial C_0}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{\partial S}{\partial C_1}\right]^2 \cdot \sigma_{C_1}^2 + \left[\frac{\partial S}{\partial m}\right]^2 \cdot \sigma_m^2$$

$$= \left[e^{-C_1 \cdot m}\right]^2 \cdot \sigma_{C_0}^2 + \left[C_0 \cdot e^{-C_1 \cdot m} \cdot (-m)\right]^2 \cdot \sigma_{C_1}^2 + \left[C_0 \cdot e^{-C_1 \cdot m} \cdot (-C_1)\right]^2 \cdot \sigma_m^2$$

$$= \left[e^{-C_1 \cdot m}\right]^2 \bullet \left[\sigma_{C_0}^2 + C_0^2 \cdot m^2 \cdot \sigma_{C_1}^2 + C_0^2 \cdot C_1^2 \cdot \sigma_m^2\right]$$

$$= \left[C_0 \cdot e^{-C_1 \cdot m}\right]^2 \bullet \left[\frac{\sigma_{C_0}^2}{C_0^2} + m^2 \cdot \sigma_{C_1}^2 + C_1^2 \cdot \sigma_m^2\right]$$

Substituting

$$S(m) = C_0 \cdot e^{-C_1 \cdot m}$$

$$\sigma_{C_0} = e^{A_0} \cdot \sigma_{A_0} = e^{A_0} \cdot \sqrt{M_{11}^{-1}} = C_0 \cdot \sqrt{M_{11}^{-1}}$$

$$\sigma_{C_1} = \sigma_{A_1} = \sqrt{M_{22}^{-1}}$$

we can write

$$(dS)^2 = S^2 \bullet \left[M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2\right]$$

or

$$dS = S \bullet \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

or

$$\frac{dS}{S} = \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

Inverse Linear

The Fitted Spillover

The coefficients of the inverse linear solution:

$$S(m) = [C_0 + C_1 \cdot m]^{-1}$$

are determined by first linearizing the equation:

$$y = \frac{1}{S(m)} = C_0 + C_1 \cdot m$$

and then solving for C_0 and C_1 by solving for the vector $\overline{b}$ in the equation

$$M \cdot \overline{b} = \overline{V}$$

in which,

$$M_{JK} = \sum_{i=1}^{N} w_i \cdot m_i^{J-1} \cdot m_i^{K-1}$$

$$\overline{b} = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix}$$

$$V_K = \sum_{i=1}^{N} w_i \cdot \frac{1}{S_i} \cdot m_i^{K-1}$$

and

$$w_i = \frac{1}{\sigma_y^2}$$

Since $$y = \frac{1}{S}$$

and $$(dy)^2 = \left(\frac{-1}{S^2}\right)^2 \cdot (d\chi)^2$$

the variance of y is given by

$$\sigma_y^2 = \frac{1}{S^4} \cdot \sigma_S^2$$

so that the weighting factor, w_i, is given by

$$w_i = \frac{S^4}{\sigma_S^2}$$

The coefficients, C_0 and C_1, are given by

$$\bar{b} = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = M^{-1} \cdot M \cdot \bar{b} = M^{-1} \cdot \bar{V}$$

The uncertainties in the coefficients C_0 and C_1 are given by

$$\sigma_{C_0}^2 = M_{11}^{-1}$$

$$\sigma_{C_1}^2 = M_{22}^{-1}$$

As before, the reduced χ^2 of the final fit is given by:

$$Z = \frac{\chi^2}{N-2}$$

where,

$$\chi^2 = \sum_{i=1}^{N} \frac{[S_i - S(m_i)]^2}{\sigma_{S_i}^2}$$

Once again, for good fits, $Z \to 1$.

The Uncertainty in the Fitted Spillover

The uncertainty in the calculated spillover is determined as follows:

Given that the spillover is calculated from the following equation:

$$S(m) = [C_0 + C_1 \cdot m]^{-1}$$

we can write

$$(dS)^2 = \left[\frac{\partial S}{\partial C_0}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{\partial S}{\partial C_1}\right]^2 \cdot \sigma_{C_1}^2 + \left[\frac{\partial S}{\partial m}\right]^2 \cdot \sigma_m^2$$

$$= \left[\frac{-1}{[C_0 + C_1 \cdot m]^2}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{-1}{[C_0 + C_1 \cdot m]^2}\right]^2 \cdot [m]^2 \cdot \sigma_{C_1}^2 + \left[\frac{-1}{[C_0 + C_1 \cdot m]^2}\right]^2 \cdot [C_1]^2 \cdot \sigma_m^2$$

$$= \left[\frac{-1}{[C_0 + C_1 \cdot m]^2}\right]^2 \bullet \left[\sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + C_1^2 \cdot \sigma_m^2\right]$$

Substituting

$$S(m) = [C_0 + C_1 \cdot m]^{-1}$$

We obtain

$$(dS)^2 = S^4 \bullet [\sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + C_1^2 \cdot \sigma_m^2]$$

Now substituting

$$\sigma_{C_0}^2 = M_{11}^{-1}$$
$$\sigma_{C_1}^2 = M_{22}^{-1}$$

We obtain

$$(dS)^2 = S^4 \bullet [M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2]$$

or

$$dS = S^2 \bullet \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

or

$$\frac{dS}{S} = S \bullet \sqrt{M_{11}^{-1} + m^2 \cdot M_{22}^{-1} + C_1^2 \cdot \sigma_m^2}$$

Inverse Quadratic

The Fitted Spillover

The coefficients of the inverse quadratic solution:

$$S(m) = [C_0 + C_1 \cdot m + C_2 \cdot m^2]^{-1}$$

are determined by first linearizing the equation:

$$y = \frac{1}{S(m)} = C_0 + C_1 \cdot m + C_2 \cdot m^2$$

and then solving for C_0, C_1, and C_2 by solving for the vector $\bar{b}$ in the equation

$$M \cdot \bar{b} = \bar{V}$$

in which,

$$M_{JK} = \sum_{i=1}^{N} w_i \cdot m_i^{J-1} \cdot m_i^{K-1}$$

$$\bar{b} = \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix}$$

$$V_K = \sum_{i=1}^{N} w_i \cdot \frac{1}{S_i} \cdot m_i^{K-1}$$

and

$$w_i = \frac{1}{\sigma_y^2}$$

Since $$y = \frac{1}{S}$$

and $$(dy)^2 = \left(\frac{-1}{S^2}\right)^2 \cdot (dS)^2$$

the variance of y is given by

$$\sigma_y^2 = \frac{1}{S^4} \cdot \sigma_S^2$$

so that the weighting factor, w_i, is given by

$$w_i = \frac{S^4}{\sigma_S^2}$$

The coefficients, C_0, C_1, and C_2, are given by

$$\bar{b} = \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix} = M^{-1} \cdot M \cdot \bar{b} = M^{-1} \cdot \bar{V}$$

The uncertainties in the coefficients C_0, C_1, and C_2 are given by

$$\sigma_{C_0}^2 = M_{11}^{-1}$$
$$\sigma_{C_1}^2 = M_{22}^{-1}$$
$$\sigma_{C_2}^2 = M_{33}^{-1}$$

With three coefficients, the reduced χ^2 of the final fit is given by:

$$Z = \frac{\chi^2}{N-3}$$

where,

$$\chi^2 = \sum_{i=1}^{N} \frac{[S_i - S(m_i)]^2}{\sigma_{S_i}^2}$$

Once again, for good fits, $Z \to 1$.

The Uncertainty in the Fitted Spillover

The uncertainty in the calculated spillover is determined as follows:

Given that the spillover is calculated from the following equation:

$$S(m) = \left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^{-1}$$

we can write

$$(dS)^2 = \left[\frac{\partial S}{\partial C_0}\right]^2 \cdot \sigma_{C_0}^2 + \left[\frac{\partial S}{\partial C_1}\right]^2 \cdot \sigma_{C_1}^2 + \left[\frac{\partial S}{\partial C_2}\right]^2 \cdot \sigma_{C_2}^2 + \left[\frac{\partial S}{\partial m}\right]^2 \cdot \sigma_m^2$$

$$= \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \cdot \sigma_{C_0}^2$$

$$+ \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \cdot (m)^2 \cdot \sigma_{C_1}^2$$

$$+ \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \cdot (m^2)^2 \cdot \sigma_{C_2}^2$$

$$+ \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \cdot (2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2$$

$$(dS)^2 = \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \bullet \left[\sigma_{C_0}^2 + (m)^2 \cdot \sigma_{C_1}^2 + (m^2)^2 \cdot \sigma_{C_2}^2 + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2\right]$$

$$= \left[\frac{-1}{\left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^2}\right]^2 \bullet \left[\sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + m^4 \cdot \sigma_{C_2}^2 + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2\right]$$

Substituting

$$S(m) = \left[C_0 + C_1 \cdot m + C_2 \cdot m^2\right]^{-1}$$

We obtain

$$(dS)^2 = S^4 \bullet \left[\sigma_{C_0}^2 + m^2 \cdot \sigma_{C_1}^2 + m^4 \cdot \sigma_{C_2}^2 + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma_m^2\right]$$

Now substituting

$$\sigma_{C_0}^2 = M_{11}^{-1}$$

$$\sigma^2_{C_1} = M^{-1}_{22}$$

$$\sigma^2_{C_2} = M^{-1}_{33}$$

We obtain

$$(dS)^2 = S^4 \bullet \left[M^{-1}_{11} + m^2 \cdot M^{-1}_{22} + m^4 \cdot M^{-1}_{33} + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma^2_m\right]$$

or

$$dS = S^2 \bullet \sqrt{M^{-1}_{11} + m^2 \cdot M^{-1}_{22} + m^4 \cdot M^{-1}_{33} + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma^2_m}$$

or

$$\frac{dS}{S} = S \bullet \sqrt{M^{-1}_{11} + m^2 \cdot M^{-1}_{22} + m^4 \cdot M^{-1}_{33} + (C_1 + 2 \cdot C_2 \cdot m)^2 \cdot \sigma^2_m}$$

7. Sample Activity

The activity is reported in units of activity per unit size (mass, volume, etc.). For certain sample types (e.g., smears), the size is pre-defined as 1 and the size units are none, causing the reported activity to simply be the total activity of the sample. The activity is calculated from the "corrected" count rate of the sample and the efficiency for the geometry of the sample. If the activity is to be reported in units other than dpm, an appropriate conversion factor is applied:

$$A_S = \frac{R_{corr'd}}{\varepsilon \cdot S} \cdot F_{AC}$$

where,

$A_S =$ the Activity of the sample (in units of activity per unit size).

$R_{corr'd} =$ the "corrected" count rate (net or gross or even Spillover corrected as determined by the analysis profile).

$\varepsilon =$ the efficiency (either simple or mass attenuated) as appropriate to the sample geometry

$S =$ the Size of the sample in the selected units. For certain sample types (e.g., smears), $S = 1$.

$F_{AC} =$ Activity Conversion Factor for the desired Activity units.

The uncertainty in the sample activity is given by

$$\sigma_{A_S} = A_S \cdot \sqrt{\frac{\sigma_R^2}{R^2} + \frac{\sigma_\varepsilon^2}{\varepsilon^2} + \frac{\sigma_S^2}{S^2}} \bullet F_{AC}$$

where,

$\sigma_{R_{corr'd}} =$ the uncertainty in the "corrected" count rate.

$\sigma_\varepsilon =$ the uncertainty in the efficiency.

$\sigma_S =$ the uncertainty in the sample size.

8. MDA

The MDA is given by

$$MDA = \frac{L_{D\_RATE}}{\varepsilon \cdot S} \bullet F_{AC}$$

where,

$F_{AC} =$ Activity Conversion Factor for the desired Activity units.

$S =$ the Size of the sample in the selected units. For certain sample types (e.g., smears), $S = 1$.

$\varepsilon =$ the efficiency (either simple or mass attenuated) as appropriate to the sample geometry

$L_{D\_RATE} =$ the "Detection Limit" (in units of rate; e.g., cpm), which is given by

$$L_{D\_RATE} = \frac{k^2}{T_S} + 2 \cdot L_{C\_RATE}$$

in which L_{C_RATE} is given by

$$L_{C\_RATE} = k \cdot \sigma_{0\_RATE} = k \cdot \sqrt{\frac{R_B}{T_S} + \sigma_{R_B}^2}$$

where

$T_S =$ the sample count time

and,

$\sigma_{R_B} =$ the **uncertainty** in the (system) BACKGROUND count rate.

$= \sqrt{\frac{R_B}{T_B}}$ for the case in which the background was determined from a single measurement

$= \sqrt{\frac{\sum_{i=1}^{N} (R_B - R_B)^2}{(N-1)}}$ if the background was determined from a set of N measurements.

This "empirical uncertainty" IS NOT YET IMPLEMENTED!

Thus,

$$L_{D\_RATE} = \frac{k^2}{T_S} + 2 \cdot k \cdot \sqrt{\frac{R_B}{T_S} + \sigma_{R_B}^2}$$

For the case in which the background was determined from a single measurement, this becomes

$$L_{D\_RATE} = \frac{k^2}{T_S} + 2 \cdot k \cdot \sqrt{\frac{R_B}{T_S} + \frac{R_B}{T_B}}$$

Substituting

$$k_\alpha = k_\beta = k = 1.645$$

we obtain

$$L_{D\_RATE} = \frac{1.645^2}{T_S} + 2 \cdot 1.645 \cdot \sqrt{\frac{R_B}{T_S} + \frac{R_B}{T_B}} = \frac{2.706}{T_S} + 3.29 \cdot \sqrt{\frac{R_B}{T_S} + \frac{R_B}{T_B}}$$

and

$$MDA = \frac{\left[\frac{2.706}{T_S} + 3.29 \cdot \sqrt{\frac{R_B}{T_S} + \frac{R_B}{T_B}}\right]}{\varepsilon \cdot S} \bullet F_{AC}$$

For the case in which the background was determined from a set of N measurements, the MDA is given by

$$MDA = \frac{\left[\frac{2.706}{T_S} + 3.29 \cdot \sqrt{\frac{R_B}{T_S} + \sigma_{R_B}^2}\right]}{\varepsilon \cdot S} \bullet F_{AC}$$

in which

$\sigma_{R_B} =$ the **uncertainty** in the (system) BACKGROUND count rate.

Appendix A: Derivation of the Activity and Count Rate Equations

This appendix includes the derivation of the activity and count rate equations for the case in which Spillover Correction is applied in the Simultaneous Mode.

A.1 Without Consideration for Method Blank Subtraction

Definitions:

$A_\beta =$ the beta activity in dpm.

$A_\alpha =$ the alpha activity in dpm.

$R_\beta =$ the GROSS beta count rate in cpm.

$R_\alpha =$ the GROSS alpha count rate in cpm.

$\varepsilon_\beta =$ the beta efficiency (either simple or mass attenuated) as appropriate to the sample geometry

$\varepsilon_\alpha =$ the alpha efficiency (either simple or mass attenuated) as appropriate to the sample geometry

$B_\beta =$ the beta (system) background count rate in cpm.

$B_\alpha =$ the alpha (system) background count rate in cpm.

$\chi_{\alpha->\beta} =$ the fractional spillover (spilldown) from alpha to beta.

$\chi_{\beta->\alpha} =$ the fractional spillover (spillup) from beta to alpha.

Spilldown:

$$\chi_{\alpha->\beta} \equiv \frac{R_{\beta\_net}}{R_{\alpha\_net}}$$

and

$$\sigma_{\chi_{\alpha->\beta}} = \chi_{\alpha->\beta} \cdot \sqrt{\left(\frac{\sigma_{R_{\beta\_net}}}{R_{\beta\_net}}\right)^2 + \left(\frac{\sigma_{R_{\alpha\_net}}}{R_{\alpha\_net}}\right)^2}$$ when determined from a single measurement[2]

Spillup:

$$\chi_{\beta->\alpha} \equiv \frac{R_{\alpha\_net}}{R_{\beta\_net}}$$

and

[2] When the spillover is determined from a set of N measurements, the uncertainty assigned to the spillover in ECLIPSE is determined from the empirical variance as described in Section 5 – Efficiency and Spillover calibration without Mass Attenuation.

$$\sigma_{\chi_{\beta->\alpha}} = \chi_{\beta->\alpha} \cdot \sqrt{\left(\frac{\sigma_{R_{\beta\_net}}}{R_{\beta\_net}}\right)^2 + \left(\frac{\sigma_{R_{\alpha\_net}}}{R_{\alpha\_net}}\right)^2}$$ when determined from a single measurement[2]

Derivation:

Rates:

The gross beta count rate includes contributions from the

- beta activity in the sample
- alpha to beta spillover
- beta background

as follows:

$$R_\beta = A_\beta \cdot \varepsilon_\beta + A_\alpha \cdot \varepsilon_\alpha \cdot \chi_{\alpha->\beta} + B_\beta$$

Similarly, the gross alpha count rate can be written as

$$R_\alpha = A_\alpha \cdot \varepsilon_\alpha + A_\beta \cdot \varepsilon_\beta \cdot \chi_{\beta->\alpha} + B_\alpha$$

We now have two equations in two unknowns (A_α and A_β), which can be rearranged as follows:

$$A_\beta \cdot \varepsilon_\beta + A_\alpha \cdot \varepsilon_\alpha \cdot \chi_{\alpha->\beta} = R_\beta - B_\beta$$

$$A_\alpha \cdot \varepsilon_\alpha + A_\beta \cdot \varepsilon_\beta \cdot \chi_{\beta->\alpha} = R_\alpha - B_\alpha$$

Or further rearranged as follows:

$$A_\beta \cdot \varepsilon_\beta + A_\alpha \cdot \varepsilon_\alpha \cdot \chi_{\alpha->\beta} = R_\beta - B_\beta$$

$$A_\beta \cdot \varepsilon_\beta \cdot \chi_{\beta->\alpha} + A_\alpha \cdot \varepsilon_\alpha = R_\alpha - B_\alpha$$

Solving these equations simultaneously yields:

$$A_\beta = \frac{\left\lfloor (R_\beta - B_\beta) - (R_\alpha - B_\alpha) \cdot \chi_{\alpha->\beta} \right\rfloor}{\varepsilon_\beta \cdot [1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}]}$$

$$A_\alpha = \frac{\left\lfloor (R_\alpha - B_\alpha) - (R_\beta - B_\beta) \cdot \chi_{\beta->\alpha} \right\rfloor}{\varepsilon_\alpha \cdot [1 - \chi_{\beta->\alpha} \cdot \chi_{\alpha->\beta}]}$$

By analogy to the general equation for total activity, namely, the count rate divided by the efficiency:

$$A = \frac{R}{\varepsilon}$$

we may write the beta and alpha activities as follows:

$$A_\beta = \frac{R_{\beta\_corrected}}{\varepsilon_\beta}$$

$$A_\alpha = \frac{R_{\alpha\_corrected}}{\varepsilon_\alpha}$$

in which

$$R_{\beta\_corrected} = \frac{\left[\left(R_\beta - B_\beta\right) - \left(R_\alpha - B_\alpha\right) \cdot \chi_{\alpha->\beta}\right]}{[1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}]}$$

and

$$R_{\alpha\_corrected} = \frac{\left[\left(R_\alpha - B_\alpha\right) - \left(R_\beta - B_\beta\right) \cdot \chi_{\beta->\alpha}\right]}{[1 - \chi_{\beta->\alpha} \cdot \chi_{\alpha->\beta}]}$$

Uncertainties

Defining:

$$R'_\alpha = R_{\alpha\_corrected}$$

and

$$R'_\beta = R_{\beta\_corrected}$$

we can write the uncertainties in the sample activities as

$$\sigma_{A_\alpha} = A_\alpha \cdot \sqrt{\frac{\sigma^2_{R'_\alpha}}{R'^2_\alpha} + \frac{\sigma^2_{\varepsilon_\alpha}}{\varepsilon^2_\alpha}}$$

$$\sigma_{A_\beta} = A_\beta \cdot \sqrt{\frac{\sigma^2_{R'_\beta}}{R'^2_\beta} + \frac{\sigma^2_{\varepsilon_\beta}}{\varepsilon^2_\beta}}$$

where,

$\varepsilon_\alpha =$ the alpha counting efficiency

$\varepsilon_\beta =$ the beta counting efficiency

$\sigma_{\varepsilon_\alpha} =$ the uncertainty in the alpha counting efficiency defined previously

$\sigma_{\varepsilon_\beta} =$ the uncertainty in the beta counting efficiency defined previously

$\sigma_{R'_\alpha} =$ the uncertainty in the corrected alpha count rate defined below

$\sigma_{R'_\beta} =$ the uncertainty in the corrected beta count rate defined below in which the uncertainties in the corrected count rates are given by (see derivation [including Method Blank subtraction] in Appendix A.2):

$$\sigma^2_{R'_\alpha} = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\alpha} + \sigma^2_{B_\alpha}\right]$$

$$+ \left[\frac{\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\beta} + \sigma^2_{B_\beta}\right]$$

$$+ \left[\frac{-R'_\beta\cdot\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\beta->\alpha}}}{\chi_{\beta->\alpha}}\right)^2$$

$$+ \left[\frac{R'_\alpha\cdot\chi_{\beta->\alpha}\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\alpha->\beta}}}{\chi_{\alpha->\beta}}\right)^2$$

$$\sigma^2_{R'_\beta} = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\beta} + \sigma^2_{B_\beta}\right]$$

$$+ \left[\frac{\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\alpha} + \sigma^2_{B_\alpha}\right]$$

$$+ \left[\frac{-R'_\alpha\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\alpha->\beta}}}{\chi_{\alpha->\beta}}\right)^2$$

$$+ \left[\frac{R'_\beta\cdot\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\beta->\alpha}}}{\chi_{\beta->\alpha}}\right)^2$$

A.2 With System Background and Method Blank Subtraction

Definitions:

$A_\beta =$ the beta activity in dpm.

$A_\alpha =$ the alpha activity in dpm.

$R_\beta =$ the GROSS beta count rate in cpm.

$R_\alpha =$ the GROSS alpha count rate in cpm.

$\varepsilon_\beta =$ the beta efficiency (either simple or mass attenuated) as appropriate to the sample geometry

$\varepsilon_\alpha =$ the alpha efficiency (either simple or mass attenuated) as appropriate to the sample geometry

$B_\beta =$ the beta (system) background count rate in cpm.

$B_\alpha =$ the alpha (system) background count rate in cpm.

$M_{\beta\_gross} =$ the gross beta count rate of the Method Blank in cpm.

$M_{\alpha\_gross} =$ the gross alpha count rate of the Method Blank in cpm.

$\chi_{\alpha->\beta} =$ the fractional spillover (spilldown) from alpha to beta.

$\chi_{\beta->\alpha} =$ the fractional spillover (spillup) from beta to alpha.

Spilldown:

$$\chi_{\alpha->\beta} \equiv \frac{R_{\beta\_net}}{R_{\alpha\_net}}$$

and

$$\sigma_{\chi_{\alpha->\beta}} = \chi_{\alpha->\beta} \cdot \sqrt{\left(\frac{\sigma_{R_{\beta\_net}}}{R_{\beta\_net}}\right)^2 + \left(\frac{\sigma_{R_{\alpha\_net}}}{R_{\alpha\_net}}\right)^2}$$ when determined from a single measurement[3]

Spillup:

$$\chi_{\beta->\alpha} \equiv \frac{R_{\alpha\_net}}{R_{\beta\_net}}$$

and

$$\sigma_{\chi_{\beta->\alpha}} = \chi_{\beta->\alpha} \cdot \sqrt{\left(\frac{\sigma_{R_{\beta\_net}}}{R_{\beta\_net}}\right)^2 + \left(\frac{\sigma_{R_{\alpha\_net}}}{R_{\alpha\_net}}\right)^2}$$ when determined from a single measurement[3]

[3] When the spillover is determined from a set of *N* measurements, the uncertainty assigned to the spillover in ECLIPSE is determined from the empirical variance as described in Section 5 – Efficiency and Spillover calibration without Mass Attenuation

Derivation:

Rates:

The gross beta count rate includes contributions from the

* beta activity in the sample
* beta activity from the Method
* beta background
* alpha to beta spillover

as follows:

$$R_\beta = A_\beta \cdot \varepsilon_\beta + M_{\beta\_net} + B_\beta + A_\alpha \cdot \varepsilon_\alpha \cdot \chi_{\alpha->\beta}$$

which can be re-written as

$$R_\beta = A_\beta \cdot \varepsilon_\beta + \delta_2 \cdot [M_{\beta\_gross} - \delta_1 \cdot B_\beta] + \delta_1 \cdot B_\beta + A_\alpha \cdot \varepsilon_\alpha \cdot \chi_{\alpha->\beta}$$

where δ_1 and δ_2 can be interpreted as follows:

δ_1 = 0 means the system background may be neglected

δ_1 = 1 means the system background is to be taken into account

δ_2 = 0 means the Method Blank contribution may be neglected

δ_2 = 1 means the Method Blank contribution is to be taken into account

Similarly, the gross alpha count rate can be written as

$$R_\alpha = A_\alpha \cdot \varepsilon_\alpha + M_{\alpha\_net} + B_\alpha + A_\beta \cdot \varepsilon_\beta \cdot \chi_{\beta->\alpha}$$

which can be re-written as

$$R_\alpha = A_\alpha \cdot \varepsilon_\alpha + \delta_2 \cdot [M_{\alpha\_gross} - \delta_1 \cdot B_\alpha] + \delta_1 \cdot B_\alpha + A_\beta \cdot \varepsilon_\beta \cdot \chi_{\beta->\alpha}$$

We now have two equations in two unknowns (A_α and A_β), which can be rearranged as follows:

$$A_\beta \cdot \varepsilon_\beta + A_\alpha \cdot \varepsilon_\alpha \cdot \chi_{\alpha->\beta} = R_\beta - \delta_2 \cdot [M_{\beta\_gross} - \delta_1 \cdot B_\beta] - \delta_1 \cdot B_\beta$$

$$A_\alpha \cdot \varepsilon_\alpha + A_\beta \cdot \varepsilon_\beta \cdot \chi_{\beta->\alpha} = R_\alpha - \delta_2 \cdot [M_{\alpha\_gross} - \delta_1 \cdot B_\alpha] - \delta_1 \cdot B_\alpha$$

Or further rearranged as follows:

$$A_\beta \cdot \varepsilon_\beta + A_\alpha \cdot \varepsilon_\alpha \cdot \chi_{\alpha->\beta} = R_\beta - \delta_2 \cdot M_{\beta\_gross} + \delta_2 \cdot \delta_1 \cdot B_\beta - \delta_1 \cdot B_\beta$$

$$A_\beta \cdot \varepsilon_\beta \cdot \chi_{\beta->\alpha} + A_\alpha \cdot \varepsilon_\alpha = R_\alpha - \delta_2 \cdot M_{\alpha\_gross} + \delta_2 \cdot \delta_1 \cdot B_\alpha - \delta_1 \cdot B_\alpha$$

and finally stated as

$$A_\beta \cdot \varepsilon_\beta + A_\alpha \cdot \varepsilon_\alpha \cdot \chi_{\alpha->\beta} = R_\beta - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_\beta \cdot (1-\delta_2)$$

$$A_\beta \cdot \varepsilon_\beta \cdot \chi_{\beta->\alpha} + A_\alpha \cdot \varepsilon_\alpha = R_\alpha - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_\alpha (1-\delta_2)$$

Solving these equations simultaneously yields:

$$A_{\beta} = \frac{\left[\left(R_{\beta} - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_{\beta} \cdot (1-\delta_2)\right) - \left(R_{\alpha} - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_{\alpha}(1-\delta_2)\right) \cdot \chi_{\alpha->\beta}\right]}{\varepsilon_{\beta} \cdot [1-\chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}]}$$

$$A_{\alpha} = \frac{\left[\left(R_{\alpha} - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_{\alpha}(1-\delta_2)\right) - \left(R_{\beta} - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_{\beta} \cdot (1-\delta_2)\right) \cdot \chi_{\beta->\alpha}\right]}{\varepsilon_{\alpha} \cdot [1-\chi_{\beta->\alpha} \cdot \chi_{\alpha->\beta}]}$$

Since the total activity is given by the count rate divided by the efficiency:

$$A = \frac{R}{\varepsilon}$$

we recognize that the spillover corrected count rates can be written as

$$R_{\beta\_corrected} = \frac{\left[\left(R_{\beta} - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_{\beta} \cdot (1-\delta_2)\right) - \left(R_{\alpha} - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_{\alpha}(1-\delta_2)\right) \cdot \chi_{\alpha->\beta}\right]}{[1-\chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}]}$$

$$R_{\alpha\_corrected} = \frac{\left[\left(R_{\alpha} - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_{\alpha}(1-\delta_2)\right) - \left(R_{\beta} - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_{\beta} \cdot (1-\delta_2)\right) \cdot \chi_{\beta->\alpha}\right]}{[1-\chi_{\beta->\alpha} \cdot \chi_{\alpha->\beta}]}$$

so that the alpha and beta activities, A_{α} and A_{β} , are given by

$$A_{\alpha} = \frac{R_{\alpha\_corrected}}{\varepsilon_{\alpha}}$$

$$A_{\beta} = \frac{R_{\beta\_corrected}}{\varepsilon_{\beta}}$$

Uncertainties

Defining:

$$R'_{\alpha} = R_{\alpha\_corrected}$$

and

$$R'_{\beta} = R_{\beta\_corrected}$$

we can write the uncertainties in the sample activities as

$$\sigma_{A_\alpha} = A_\alpha \cdot \sqrt{\frac{\sigma^2_{R'_\alpha}}{R'^2_\alpha} + \frac{\sigma^2_{\varepsilon_\alpha}}{\varepsilon^2_\alpha}}$$

$$\sigma_{A_\beta} = A_\beta \cdot \sqrt{\frac{\sigma^2_{R'_\beta}}{R'^2_\beta} + \frac{\sigma^2_{\varepsilon_\beta}}{\varepsilon^2_\beta}}$$

where,

$\varepsilon_\alpha =$ the alpha counting efficiency

$\varepsilon_\beta =$ the beta counting efficiency

$\sigma_{\varepsilon_\alpha} =$ the uncertainty in the alpha counting efficiency defined previously

$\sigma_{\varepsilon_\beta} =$ the uncertainty in the beta counting efficiency defined previously

$\sigma_{R'_\alpha} =$ the uncertainty in the corrected alpha count rate defined below

$\sigma_{R'_\beta} =$ the uncertainty in the corrected beta count rate defined below

in which the uncertainties in the corrected count rates are given by

$$\sigma^2_{R'_\alpha} = \left[\frac{1}{[1-\chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\alpha} + (\delta_2)^2 \cdot \sigma^2_{M_{\alpha\_gross}} + (\delta_1)^2 \cdot (1-\delta_2)^2 \cdot \sigma^2_{B_\alpha}\right]$$

$$+ \left[\frac{\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\beta} + (\delta_2)^2 \cdot \sigma^2_{M_{\beta\_gross}} + (\delta_1)^2 \cdot (1-\delta_2)^2 \cdot \sigma^2_{B_\beta}\right]$$

$$+ \left[\frac{-R'_\beta \cdot \chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\beta->\alpha}}}{\chi_{\beta->\alpha}}\right)^2$$

$$+ \left[\frac{R'_\alpha \cdot \chi_{\beta->\alpha} \cdot \chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}]}\right]^2 \cdot \left(\frac{\sigma_{\chi_{\alpha->\beta}}}{\chi_{\alpha->\beta}}\right)^2$$

$$\sigma_{R'_\beta}^2 = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma_{R_\beta}^2 + (\delta_2)^2\cdot\sigma_{M_{\beta\_gross}}^2 + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma_{B_\beta}^2\right]$$

$$+ \left[\frac{\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma_{R_\alpha}^2 + (\delta_2)^2\cdot\sigma_{M_{\alpha\_gross}}^2 + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma_{B_\alpha}^2\right]$$

$$+ \left[\frac{-R'_\alpha\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2\cdot\left(\frac{\sigma_{\chi_{\alpha->\beta}}}{\chi_{\alpha->\beta}}\right)^2$$

$$+ \left[\frac{R'_\beta\cdot\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2\cdot\left(\frac{\sigma_{\chi_{\beta->\alpha}}}{\chi_{\beta->\alpha}}\right)^2$$

DERIVATION:

Noting our previous definitions:

$$R'_\alpha = R_{\alpha\_corrected}$$

$$R'_\beta = R_{\beta\_corrected}$$

we may write the corrected count rates as follows:

$$R'_\alpha = [(R_\alpha - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_\alpha \cdot (1-\delta_2)) - (R_\beta - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_\beta (1-\delta_2)) \cdot \chi_{\beta->\alpha}] \bullet [1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}$$

$$R'_\beta = [(R_\beta - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_\beta \cdot (1-\delta_2)) - (R_\alpha - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_\alpha (1-\delta_2)) \cdot \chi_{\alpha->\beta}] \bullet [1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}$$

Carrying out the derivation for the uncertainty in the corrected beta count rate, and noting that a similar result applies to the alpha case, we proceed as follows:

$$(dR'_\beta)^2 = \left[\frac{\partial R'_\beta}{\partial R_\beta}\right]^2 \cdot \sigma^2_{R_\beta} + \left[\frac{\partial R'_\beta}{\partial M_{\beta\_gross}}\right]^2 \cdot \sigma^2_{M_{\beta\_gross}} + \left[\frac{\partial R'_\beta}{\partial B_\beta}\right]^2 \cdot \sigma^2_{B_\beta}$$

$$+ \quad \left[\frac{\partial R'_\beta}{\partial R_\alpha}\right]^2 \cdot \sigma^2_{R_\alpha} + \left[\frac{\partial R'_\beta}{\partial M_{\alpha\_gross}}\right]^2 \cdot \sigma^2_{M_{\alpha\_gross}} + \left[\frac{\partial R'_\beta}{\partial B_\alpha}\right]^2 \cdot \sigma^2_{B_\alpha}$$

$$+ \quad \left[\frac{\partial R'_\beta}{\partial \chi_{\alpha->\beta}}\right]^2 \cdot \sigma^2_{\chi_{\alpha->\beta}}$$

$$+ \quad \left[\frac{\partial R'_\beta}{\partial \chi_{\beta->\alpha}}\right]^2 \cdot \sigma^2_{\chi_{\beta->\alpha}}$$

$$(dR'_{\beta})^2 = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{R_\beta} + \left[\frac{-\delta_2}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{M_{\beta\_gross}} + \left[\frac{-\delta_1\cdot(1-\delta_2)}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{B_\beta}$$

$$+ \left[\frac{-\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{R_\alpha} + \left[\frac{\delta_2\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{M_{\alpha\_gross}} + \left[\frac{\delta_1\cdot(1-\delta_2)\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{B_\alpha}$$

[4]
$$+ \left[\frac{-R'_\alpha}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{\chi_{\alpha->\beta}}$$

[5]
$$+ \left[\frac{R'_\beta\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{\chi_{\beta->\alpha}}$$

Extract the common factors from lines 1 and 2:

$$(dR'_{\beta})^2 = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\beta} + (-\delta_2)^2\cdot\sigma^2_{M_{\beta\_gross}} + (-\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\beta}\right]$$

$$+ \left[\frac{\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[(-1)^2\cdot\sigma^2_{R_\alpha} + (\delta_2)^2\cdot\sigma^2_{M_{\alpha\_gross}} + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\alpha}\right]$$

$$+ \left[\frac{-R'_\alpha}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{\chi_{\alpha->\beta}}$$

$$+ \left[\frac{R'_\beta\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \cdot \sigma^2_{\chi_{\beta->\alpha}}$$

[4] See section A.3 - Differentiation, Rearrangement, and Simplification of Partial Derivative for details.

[5] See section A.3 - Differentiation, Rearrangement, and Simplification of Partial Derivative for details.

Multiply lines 3 and 4 by $\left(\frac{\chi_{\alpha->\beta}}{\chi_{\alpha->\beta}}\right)^2$ and $\left(\frac{\chi_{\beta->\alpha}}{\chi_{\beta->\alpha}}\right)^2$, respectively:

$$(dR'_\beta)^2 = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\beta} + (-\delta_2)^2\cdot\sigma^2_{M_{\beta\_gross}} + (-\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\beta}\right]$$

$$+ \left[\frac{\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[(-1)^2\cdot\sigma^2_{R_\alpha} + (\delta_2)^2\cdot\sigma^2_{M_{\alpha\_gross}} + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\alpha}\right]$$

$$+ \left[\frac{-R'_\alpha\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2\cdot\left(\frac{\sigma_{\chi_{\alpha->\beta}}}{\chi_{\alpha->\beta}}\right)^2$$

$$+ \left[\frac{R'_\beta\cdot\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2\cdot\left(\frac{\sigma_{\chi_{\beta->\alpha}}}{\chi_{\beta->\alpha}}\right)^2$$

Noting that $(-1)^2 = 1$ (in line 2) and does not need to be stated explicitly, and deleting the unnecessary negative signs from line 1 yields:

$$(dR'_\beta)^2 = \left[\frac{1}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\beta} + (\delta_2)^2\cdot\sigma^2_{M_{\beta\_gross}} + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\beta}\right]$$

$$+ \left[\frac{\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2 \bullet \left[\sigma^2_{R_\alpha} + (\delta_2)^2\cdot\sigma^2_{M_{\alpha\_gross}} + (\delta_1)^2\cdot(1-\delta_2)^2\cdot\sigma^2_{B_\alpha}\right]$$

$$+ \left[\frac{-R'_\alpha\cdot\chi_{\alpha->\beta}}{[1-\chi_{\alpha->\beta}\bullet\chi_{\beta->\alpha}]}\right]^2\cdot\left(\frac{\sigma_{\chi_{\alpha->\beta}}}{\chi_{\alpha->\beta}}\right)^2$$

$$+ \left[\frac{R'_\beta\cdot\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}}{[1-\chi_{\alpha->\beta}\cdot\chi_{\beta->\alpha}]}\right]^2\cdot\left(\frac{\sigma_{\chi_{\beta->\alpha}}}{\chi_{\beta->\alpha}}\right)^2$$

A.3 Differentiation, Rearrangement, and Simplification of Partial Derivatives

Noting that R'_α and R'_β are given by

$$R'_\alpha = \left[\left(R_\alpha - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_\alpha \cdot (1-\delta_2)\right) - \left(R_\beta - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_\beta (1-\delta_2)\right) \cdot \chi_{\beta->\alpha}\right] \bullet [1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}$$

$$R'_\beta = \left[\left(R_\beta - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_\beta \cdot (1-\delta_2)\right) - \left(R_\alpha - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_\alpha (1-\delta_2)\right) \cdot \chi_{\alpha->\beta}\right] \bullet [1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}$$

we can simplify the notation with the following definitions:

$$f = \left(R_\beta - \delta_2 \cdot M_{\beta\_gross} - \delta_1 \cdot B_\beta \cdot (1-\delta_2)\right)$$
$$g = \left(R_\alpha - \delta_2 \cdot M_{\alpha\_gross} - \delta_1 \cdot B_\alpha (1-\delta_2)\right)$$

so that we may write

$$R'_\alpha = \frac{\left(g - f \cdot \chi_{\beta->\alpha}\right)}{\left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)} = \left(g - f \cdot \chi_{\beta->\alpha}\right) \bullet \left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)^{-1} = w \bullet v$$

and

$$R'_\beta = \frac{\left(f - g \cdot \chi_{\alpha->\beta}\right)}{\left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)} = \left(f - g \cdot \chi_{\alpha->\beta}\right) \bullet \left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)^{-1} = u \bullet v$$

in which

$$w = \left(g - f \cdot \chi_{\beta->\alpha}\right)$$
$$u = \left(f - g \cdot \chi_{\alpha->\beta}\right)$$
$$v = \left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)^{-1}$$

which simplify the required differentiations. Since the derivation of the uncertainty in the corrected count rate is being carried out for the beta case, we proceed as follows:

$$u = \left(f - g \cdot \chi_{\alpha->\beta}\right)$$

$$\frac{\partial u}{\partial \chi_{\alpha->\beta}} = (-g)$$

$$\frac{\partial u}{\partial \chi_{\beta->\alpha}} = 0$$

$$v = \left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)^{-1}$$

$$\frac{\partial v}{\partial \chi_{\alpha->\beta}} = (-1) \cdot \left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{-2} \cdot \left(-\chi_{\beta->\alpha}\right)$$

$$= \frac{\chi_{\beta->\alpha}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}}$$

$$\frac{\partial v}{\partial \chi_{\beta->\alpha}} = (-1) \cdot \left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{-2} \cdot \left(-\chi_{\alpha->\beta}\right)$$

$$= \frac{\chi_{\alpha->\beta}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}}$$

We may now write:

$$\frac{\partial R'_{\beta}}{\partial \chi_{\alpha->\beta}} = u \cdot \frac{\partial v}{\partial \chi_{\alpha->\beta}} + v \cdot \frac{\partial u}{\partial \chi_{\alpha->\beta}}$$

$$= \left(f - g \cdot \chi_{\alpha->\beta}\right) \cdot \frac{\chi_{\beta->\alpha}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}} + \left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)^{-1} \cdot (-g)$$

$$= \frac{\left(f - g \cdot \chi_{\alpha->\beta}\right) \cdot \chi_{\beta->\alpha}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}} + \frac{(-g)}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]}$$

$$= \frac{\left(f - g \cdot \chi_{\alpha->\beta}\right) \cdot \chi_{\beta->\alpha}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}} + \frac{(-g) \cdot \left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}}$$

$$= \frac{f \cdot \chi_{\beta->\alpha} - g \cdot \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}} + \frac{(-g) + g \cdot \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}}$$

$$= \frac{f \cdot \chi_{\beta->\alpha} - g}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}}$$

[6] $$= \frac{-R'_{\alpha}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]}$$

---

[6] By substituting $R'_{\alpha} = \frac{\left(g - f \cdot \chi_{\beta->\alpha}\right)}{\left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)}$ or more specifically $-R'_{\alpha} = \frac{\left(f \cdot \chi_{\beta->\alpha} - g\right)}{\left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)}$

and

$$\frac{\partial R'_{\beta}}{\partial \chi_{\beta->\alpha}} = u \cdot \frac{\partial v}{\partial \chi_{\beta->\alpha}} + v \cdot \frac{\partial u}{\partial \chi_{\beta->\alpha}}$$

$$= \left(f - g \cdot \chi_{\alpha->\beta}\right) \cdot \frac{\chi_{\alpha->\beta}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}} + \left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)^{-1} \cdot 0$$

$$= \frac{\left(f - g \cdot \chi_{\alpha->\beta}\right) \cdot \chi_{\alpha->\beta}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]^{2}}$$

[7] $$= \frac{R'_{\beta} \cdot \chi_{\alpha->\beta}}{\left[1 - \chi_{\alpha->\beta} \bullet \chi_{\beta->\alpha}\right]}$$

[7] By substituting $R'_{\beta} = \frac{\left(f - g \cdot \chi_{\alpha->\beta}\right)}{\left(1 - \chi_{\alpha->\beta} \cdot \chi_{\beta->\alpha}\right)}$

Warranty

This warranty covers Canberra hardware and software shipped to customers within the United States. For hardware and software shipped outside the United States, a similar warranty is provided by Canberra's local representative.

DOMESTIC WARRANTY

Equipment manufactured by Canberra is warranted against defects in materials and workmanship for one year from the date of shipment.

Canberra warrants proper operation of its software only when used with software and hardware supplied by Canberra and warrants software media to be free from defects for 90 days from the date of shipment.

If defects are discovered within 90 days of the time you receive your order, Canberra will pay transportation costs. After the first 90 days, you will have to pay the transportation costs.

LIMITATIONS

Upon notification of defects in the software media or hardware, Canberra will repair or replace the defective items at its discretion.

THIS IS THE ONLY WARRANTY PROVIDED BY CANBERRA; THERE ARE NO OTHER WARRANTIES, EXPRESSED OR IMPLIED. ALL WARRANTIES OF MERCHANTABILITY AND FITNESS FOR AN INTENDED PURPOSE ARE EXCLUDED. CANBERRA SHALL HAVE NO LIABILITY FOR ANY SPECIAL, INDIRECT OR CONSEQUENTIAL DAMAGES CAUSED BY FAILURE OF ANY EQUIPMENT OR SOFTWARE MANUFACTURED BY CANBERRA.

EXCLUSIONS

This warranty does not cover equipment which has been modified without Canberra's written permission or which has been subjected to unusual physical or electrical stress as determined by Canberra's Service Personnel.

Canberra is under no obligation to provide warranty service if adjustment or repair is required because of damage caused by other than ordinary use or if the equipment is serviced or repaired, or if an attempt is made to service or repair the equipment, by other than Canberra personnel without the prior approval of Canberra.

This warranty does not cover detector damage due to neutrons or heavy charged particles or from physical abuse. Failure of beryllium, carbon composite, or polymer windows or of windowless detectors caused by physical or chemical damage from the environment is not covered by warranty.

Canberra is not responsible for damage sustained in transit. Examine shipments carefully when you receive them for evidence of damage caused in transit. If damage is found, notify Canberra and the carrier immediately. Keep all packages, materials and documents, including your freight bill, invoice and packing list.

Revised 04/97

Software License

You have purchased the license to use Canberra software, not the software itself. Since title to the software remains with Canberra, you may not sell or transfer the software. This license allows you to use the software on only one computer at a time. You must get Canberra's written permission for any exception to this license.

BACKUP COPIES

Canberra's software is protected by United States Copyright Law and by International Copyright Treaties. You have Canberra's express permission to make one archival copy of this software for backup protection. You may not copy Canberra software or any part of it for any other purpose.